Review Exercises - - Math 251

Boggess - - Fall, 1997

General Statement. You should have a good working knowledge of the following topics from Math 151, 152: limits, differentiation rules, derivative as a rate of change, basic curve sketching with calculus, integration rules, the integral as a sum, vectors in 2 and 3 dimensions, lines and planes in 3-space, functions of several variables and their graphs, partial derivatives and gradients, and the tangent plane to a graph of a function of two variables. The following review exercises should help you review some of these important concepts from Math 152.

Integrals.

1. Do the following integrals

$$\int \frac{x\,dx}{\sqrt{1+x^2}} \qquad \int xe^{2x}\,dx \qquad \int_3^5 \frac{dx}{x^2-4}$$

2. Determine whether or not the following integrals converge or diverge

$$\int_0^\infty \frac{dx}{\sqrt{x^3+x}} \qquad \int_0^1 \frac{dx}{x\sqrt{x}}$$

3. Set up the integral that computes the area between the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

4. Set up the integral that computes the volume of the solid with a base given by the unit circle and with cross sections perpendicular to the base given by equilateral triangles.

5. A heavy rope, 50 feet long, weighs 0.5 lb/ft and hangs over the edge of building 120 feet high. How much work is done in pulling the rope to the top of the building?

Vectors, lines and planes in 3-space

6. Find the angle between the vectors $\langle 1, 2, 2\rangle$ and $\langle 3, 4, 0\rangle$.

7. Find the equation of the line that is perpendicular to the plane $2x + y - 3z = 5$ and passing through the point $(4, -1, 2)$. Express the answer in both parametric and symmetric form.

8. Find the equation of the plane that passes through the points $(1, 0, -3)$, $(0, -2, -4)$ and $(4, 1, 6)$.

9. Find the equation of the tangent line to the curve $x = t$, $y = t^2$, $z = t^3$ at the point $(1, 1, 1)$. Express the answer in both parametric and symmetric form.

10. Find the distance from the point (2,8,5) to the plane $x - 2y - 2z = 1$.

Functions of several variables and their graphs

11. Graph the following surfaces

$$z = x^2 - 2y^2 \qquad z^2 + 2x^2 + 4y^2 = 16 \qquad z^2 - x^2 + y^2 = 4$$

12. Describe the level curves of the following functions of two variables. (Recall that a level curve of a function $f(x, y)$ is a set in the plane of the form $\{(x, y);\ f(x, y) = k\}$ where k is a constant).

$$f(x, y) = x^2 - y^2 \qquad f(x, y) = \frac{x^2 + y^2}{x}$$

13. Describe the level surfaces of the following functions of three variables (Recall that a level surface of a function $f(x, y, z)$ is a set in the 3-space of the form $\{(x, y, z);\ f(x, y, z) = k\}$ where k is a constant).

$$f(x, y, z) = x^2 - y^2 - z^2 \qquad f(x, y, z) = 3x - 2y + 4z$$

Partial Derivatives, gradients and tangent planes

14. Find the partial derivatives of the following functions with respect to x and y

$$f(x, y) = x\cos(x^2 y) \qquad f(x, y) = e^{xy}\sqrt{x^2 + y^2}$$

15. Find a unit vector that points in the direction of maximum increase of the given function at the given point (recall that the gradient points in the direction of maximum increase).

$$f(x, y) = x^2 y^3 \ \text{ at } \ (1, 2) \qquad f(x, y, z) = x^2 + y^3 + z^4 \ \text{ at } \ (-1, 3, 2)$$

16. Find the equation of the tangent plane to the given surface at the given point.

$$z = -x^2 y^3 \ \text{ at } \ (1, 1, -1) \qquad x^2 + 2y^2 - z^2 = 8 \ \text{ at } \ (1, 2, 1)$$

17. Indicate the possible directions of the gradient vectors at various points on the following grid that represents the level sets (or contour map) of a function of two variables. What missing information regarding the grid would be helpful in order to more accurately determine the gradient vectors?