Math 222 - - Exam I

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1. Write the vector

$$\begin{pmatrix} -6 \\ 10 \\ -19 \end{pmatrix}$$

as a linear combination of the vectors

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad v_2 = \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix} \quad v_3 = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$$

Is this linear combination unique? (12 points)

2. Find the elementary matrix E such that $EA = B$ where

$$A = \begin{pmatrix} 3 & 6 & -9 \\ 2 & 1 & 0 \\ 2 & -1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 3 & 6 & -9 \\ 2 & 1 & 0 \\ 0 & -5 & 10 \end{pmatrix}$$

(12 points)

3. What is the dimension of the nullspace of the matrix

$$\begin{pmatrix} 1 & 2 & -1 & 3 & 4 \\ 0 & 0 & 1 & -2 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Justify your answer. (12 points)

4. Use elementary row operations to reduce the following matrix to an upper triangular matrix. In the process, compute the determinant.

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -3 & 1 & 2 \end{pmatrix}$$

(12 points)

5. For each of the following sets, state whether or not it is a subspace of the given vector space. Give short reasons for your answers.

   (a) $\{(x, y, z) \in R^3;\ x - 2y + z = 0\}$ R^3 is the vector space.

   (b) $\{(x, y, z) \in R^3;\ x^2 - 2y^2 + z^2 = 0\}$ R^3 is the vector space.

   (c)

$$\left\{ \alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \end{pmatrix};\ \alpha, \beta \text{ are any real numbers} \right\}$$

   The vector space is R^3.

   (d) The set of polynomials of degree 2 with nonnegative coefficients - - i.e, $\{a_2x^2 + a_1x + a_0;\ a_2,\ a_1,\ a_0$ are all nonnegative$\}$. The vector space is the space of all continuous functions defined on the real number line.

   (e) The set of all differentiable functions f, with $f'(0) = 0$. The vector space is the space of all continuous functions defined on the real number line.

(5 points each part)

6. Give a careful proof of the following: suppose A is an $n \times n$ matrix; show that A is a nonsingular matrix if and only if the nullspace of A is $\{\mathbf{0}\}$. (12 points)

7. If A is a nonsingular matrix, show that $\det(A^{-1}) = 1/\det A$. *Hint: use the equation* $AA^{-1} = I$. (12 points)