

## Student Review Problems for Final Exam

### Spring 2000

1. How long will it take to triple your money if it is placed into an account at 12% compounded weekly? (Assume no other withdrawals or deposits are made during this time.)
2. A bank advertises its nominal rate to be 8% compounded annually. A second bank advertises its nominal rate to be 7.8% compounded daily. Which bank has the higher effective yield? How much more money will be made from the higher yield bank if you invested \$1000 for one year?
3. A car purchased for \$20,000 can be depreciated to \$5,000 in 10 years, using straight line depreciation. How long after the car is purchased will it be worth \$10,000?
4. If a car is bought for \$20,000 and depreciates to \$0 in 10 years, how long after the car is purchased will it be worth \$6,000 using straight line depreciation?
5. A company sells a certain model of car for \$12,500. If  $x$  is the number of cars sold, what is the revenue as a function of  $x$ ?
6. An item sells for \$5.25 each. Fixed costs are \$100. Profits are \$100 when 50 items are made and sold. Find the revenue, cost, and profit equations.
7.  $C(x) = 3x + 6$ ,  $R(x) = 6x$ . Find the break-even quantity.
8.  $P(x) = -4.5x^2 + 12x + 25.3$ . Find the break-even quantity and the value of  $x$  when the profit is maximized.
9. Section 1.4 #47 (pg.49)
10. Suppose 800 videos are sold at a price of \$24 per video and 1600 videos are sold if the price is \$20 per video. It costs \$8 to produce each video, with no fixed costs.
  - (a) Find the linear demand equation for price,  $p$ , as a function of quantity,  $x$ .
  - (b) How many videos would need to be produced and sold to max profit?
  - (c) How many videos would need to be produced and sold to break even?
11. A young entrepreneur wishes to paint and sell T-shirts. He invests \$200 in paint per month. This month, his client will pay \$1050 for 70 shirts or \$1000 for 50 shirts. He buys shirts from a discount store at \$5.13 each.
  - (a) Find the demand equation.
  - (b) Find the profit equation.
  - (c) How many shirts could he negotiate to sell to maximize his profits?
  - (d) What is the least amount of shirts he must sell to avoid debt?
12. The following table represents the cost of living for 1 year of college.  $x$  is the month ( $1 = \text{August}$ )  $y$  is money in dollars spent each month

Months( $x$ )	1	2	3	4	5	6	7	8	9	10	11	12
Money( $y$ )	352	106	109	96	200	150	83	97	91	163	44	52

Is there a model which can accurately depict this situation? If so, give the model and correlation coefficient. If not, explain why not.

13. Recently a group of professors at A&M found a relationship between temperature (in  $^{\circ}F$ ) and corn production per plant. The data is shown below

Temp(x)	61	64	66	68	70	72	74	76	78	80
Production(y)	11	13.2	15	18.1	21	25	27.3	30.1	26.4	25.7

Find the best mathematical model for the data and explain your answer.

14. The following table gives the relationship between the number of Lamborghini Diablos sold and their price(in thousands of dollars):

Price(x)	100	200	300	400	500	600
Cars sold(y)	531	397	158	147	121	98

- (a) Which model (linear, quadratic, or exponential) best explains this data? Write your model to 4 decimal places.
- (b) Use the unrounded model to predict the number of Lamborghinis sold if the company charges \$350,000 for them.
- (c) At what price will 200 people purchase Diablos?
15. Given  $f(x) = x + 3$ ,  $g(x) = 2x^2$ , find

- (a)  $(f \circ g)(x)$  and its domain
- (b)  $(f \circ g)(1)$

16. Given  $f(x) = x^2 + x + 1$ ,  $g(x) = x^3 + 7$ , find

- (a)  $(f \circ g)(x)$
- (b)  $(g \circ f)(x)$
- (c)  $(f \circ g)(5)$
- (d)  $(g \circ f)(1)$

17. Simplify  $3e^{\ln \sqrt{2}}$  exactly.

18. Write the following in terms of  $\log_a x, \log_a y, \log_a z$ :  $\log_a \left( \frac{x^2 y^3}{x \sqrt{z}} \right)$

19. Write the following in terms of  $\log x, \log y, \log z$ :  $\log \left( \frac{\sqrt{xy}}{z} \right)$

20. Solve the following for  $x$  exactly:

- (a)  $\log(\log_2 6x) = 2$
- (b)  $(\log 4 - \log 2) + \log x = \log 12$

21. Find the following limits:

- (a)  $\lim_{x \rightarrow \infty} \frac{e^2 - 1}{\sqrt{x} - 3}$
- (b)  $\lim_{x \rightarrow \infty} \frac{3 + e^x}{9e^x - 16}$

22. Using the limit definition of derivative, find the derivative of  $f(x) = x^3 + 4$

23. Find the derivative of  $f(x) = 3x^3 - 5$  using the definition of derivative. Use this to find the equation of the tangent line at  $x = 3$ .

24. Find the IROC of  $f(x) = 5x^2 + 3x + 5$  at  $x = 4$ .
25. The revenue function for a firm is given by  $R(x) = \sqrt{2x + 5}$ . Find the IROC when  $x$  is 2 and when it is 10. Interpret your answers.
26. Find the derivative of the following:
- $f(x) = (x + 2)^{1/2}(2x^3)$
  - $f(x) = 2x^3 + e^{3x+1} + \pi^3 + 3$
  - $f(x) = \frac{x^5+5}{x^5-5}$
  - $f(x) = \ln\left(\frac{6x^3-5x^2-1}{e^{4x}}\right)$
  - $f(x) = [e^{(2x^2+e^x+\pi+8x)} + 6x + \frac{4}{x}][\ln(3x^5 + 5x + 4)]^{1/2}$
  - $f(x) = e^{(5x^2+3x-7)} + 14^{(-12x+3)} + e^{21}$
  - $f(x) = \frac{\left(\frac{\sqrt{\sqrt{9x^2+3x^3-9x}+\sqrt{9x+5x^{10}}}}{3x^2-1}\right)}{4x \ln 2}$
27. Find the second derivative of  $f(x) = x^5 + 4x^3 - 3x^2 + 2x + 800$ .
28. Give the equation of the transformation of  $f(x) = |x^3|$  contracted by 0.5 and shifted up by 2.
29. Knowing the graph of  $f(x)$ , describe how the graphs below would be different.
- $-f(x + 8) - 9$
  - $-3f(x + 5) - 0.5$
30. Is  $f(x) = 8x^6 - 2x^4 + 2$  continuous?
31. Sketch a graph of the following and find where it is not continuous.  $f(x) = \begin{cases} -x + 2 & , x < 1 \\ x^2 & , x > 1 \\ 1/x & , x = 1 \end{cases}$
32. What are the asymptotes (horizontal and vertical), if any, of  $f(x) = \frac{20x^4+5x}{10x^4+2}$ ?
33. Sketch  $6x + 2x^2 - x^3$
34.  $f(x) = \frac{1}{1+e^{x^2}}$  Find all critical values, intervals of increasing and decreasing, all relative extremum, and sketch.
35. Sketch a graph of a function that satisfies the following conditions:
- inflection points at  $(-2,0)$  and  $(2,0)$
  - HA:  $y=0$
  - VA:  $x=-3$ ,  $x=3$ , and  $x=0$
  - $f'(x) > 0$  on  $(0, 3)$  and  $(3, \infty)$
  - $f'(x) < 0$  on  $(-\infty, -3)$  and  $(-3, 0)$
  - $f''(x) > 0$  on  $(-3, -2)$ , and  $(2, 3)$
  - $f''(x) < 0$  on  $(-\infty, -3)$ ,  $(-2, 0)$ ,  $(0, 2)$ , and  $(3, \infty)$
36. Find the points of absolute extremum of
- $2x^3 - 3x^2 + 2$  on  $(-\infty, \infty)$
  - $x^4 + 2x^3 + 6x^2 + 12x + 4$  on  $[-1, 3]$

37. You are given a piece of cardboard with the following dimensions. If square corners are cut so that the sides can be folded up to form a box with no top, what will be the resulting dimensions of the box with maximum volume? (Remember to find intervals for where the solution may be found.)
- 4-inch by 6-inch
  - 10-inch by 10-inch
  - 6-inch by 6-inch
  - If you place the box from (c) inside the box from (b), what will the resulting volume be?
38. A rectangular box has a square base with a volume of 128 cubic inches. If the bottom costs three times as much (per area) as the sides and the top, find the dimensions of the box that minimize cost.
39. Using calculus (and remembering to find intervals of possible solutions), find two non-negative numbers,  $x$  and  $y$ , such that  $x + y = 40$  and
- the sum of their squares is minimized.
  - the sum of their squares is maximized.
40. A farmer cuts a rectangular pasture in half with a fence parallel to the outside fences. What's the least amount of fence he can use if he has to enclose
- 234 square feet?
  - 216 square feet?
41. Integrate the following:
- $\int 2e^{2x+1} dx$
  - $\int (4x^{-1} + 5e^x + 6x) dx$
  - $\int 4x\sqrt{x^2 - 4} dx$
  - $\int \frac{(8x - 7)\ln(4x^2 - 7x)}{4x^2 - 7x} dx$
  - $\int (1 + \frac{12}{x}) dx$
  - $\int 3x^2(2x^3 - 1)^4 dx$
42. Integrate the following:
- $\int_0^6 \frac{1}{\sqrt{3x+1}} dx$
  - $\int_0^1 4x^3\sqrt{12+x^2} dx$
43. Using the fundamental theorem of calculus, find  $\int_2^5 \frac{2x + \sqrt{x}}{x} dx$
44. Find an upper and lower estimate of the definite integral  $\int_0^1 (x^2 + 2x + 3) dx$
45. Find the demand function for a candy cane manufacturer if marginal demand, in dollars, is given by  $p'(x) = -x^{-5/2}$ , where  $x$  is the number of thousands of candy canes sold. Assume  $p(1) = 100$ .
46. Find the revenue function if the marginal revenue is given by  $R'(x) = -8xe^{-x^2+4}$

47. The marginal revenue for a company making Aggie bumper stickers is  $8x(10-x^2)$ . Find the revenue function where  $x$  is the number of hundreds of bumper stickers sold.
48. Find the revenue function for a shoe manufacturer if the marginal revenue, in dollars, is given by  $200 - 0.26x$ , where  $x$  is the number of shoes sold.
49. Find the cost function for a strawberry soda manufacturer if the marginal cost, in dollars, is given by  $\frac{12s+12}{s^2+2s+1}$ , where  $s$  is the number of cases of strawberry soda produced and fixed costs are \$2500. What is the cost of producing five cases?
50. A jewelry store finds that its marginal revenue, in dollars, is given by  $32x^2 + e^x$ , where  $x$  is the number of thousands of diamonds sold. Find the revenue from selling 8000 diamonds.
51. Find the cost function for a shoe store if its marginal cost in dollars is  $260 - 0.007e^{2x+1}$ , where  $x$  is the number of cases of shoes produced. Assume a fixed cost of \$65.
52. A mosquito population is increasing at a rate of  $p'(t) = 1500e^{t^2}$ , where  $t$  is the number of years since the beginning of 1980. What is the population increase from the beginning of 1980 to the beginning of 1989?
53. The number of students enrolled at A&M increases annually at a rate given by  $S'(t) = 200e^{0.03t}$ , where  $t$  is the number of years since the beginning of 1997. Find the change in enrollment from the beginning of 1998 to the end of 2000.
54. Find the area enclosed by  $-x^4 + 3x^3 - 2x + 1$  and  $(x - 1)^3$  on the interval  $[-1, 4]$ .
55. Given the following sets of demand and supply functions, find the equilibrium point, consumers' surplus, producers' surplus, and sketch each on a graph.
- (a)  $D(x) = 15 - 0.25x^2, S(x) = 5x^2 + 0.32x$
- (b)  $D(x) = 24 + x - 2x^2, S(x) = x^2 + 16$
- (c)  $D(x) = 50 - 2x^2, S(x) = x^2 + 4.5$