Rules of Integration

- $\int e^x\,dx = e^x + C$
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$
- $\int k\,dx = kx + C$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\int k f(x)\,dx = k \int f(x)\,dx$$

1. Integrate the following:

Ck: $y' = \underbrace{2x^3 - 4x + 3 + 0}$

(a) $\int (2x^3 - 4x + 3)\, dx = \int 2x^3 dx - \int 4x\, dx + \int 3\, dx$

$= 2\int x^3 dx - 4\int x^1 dx + \int 3\, dx$

$= 2\left(\frac{x^{3+1}}{3+1}\right) - 4\left(\frac{x^{1+1}}{1+1}\right) + 3x + C$

$= \frac{2}{4}x^4 - \frac{4}{2}x^2 + 3x + C = \underbrace{\boxed{\frac{1}{2}x^4 - 2x^2 + 3x + C}}_{y}$

(b) $\int \left(\frac{1}{\sqrt{t}} - \sqrt[3]{t^4}\right) dt = \int \left(t^{-1/2} - t^{4/3}\right) dt$

$= \frac{t^{1/2}}{1/2} - \frac{t^{7/3}}{7/3} + c = \boxed{2t^{1/2} - \frac{3}{7}t^{7/3} + c}$

(c) $\int (y^{1.4} - 5y^{3/2})\, dy = \frac{y^{2.4}}{2.4} - 5\left(\frac{y^{5/2}}{5/2}\right) + C$

$5 \cdot \frac{2}{5} = 2$

$= \boxed{\frac{5}{12}y^{2.4} - 2y^{5/2} + c}$

(d) $\int (e^x + \frac{1}{x^2} - e^2)\, dx = \int (e^x + x^{-2} - e^2)\, dx$ a constant

$= e^x + \left(\frac{x^{-1}}{-1}\right) - e^2 x + C$

$= \boxed{e^x - x^{-1} - e^2 x + C}$

(e) $\int (3\sqrt[5]{x} - ex + x^7)\, dx = \int (\boxed{3} x^{1/5} - \boxed{e} x^1 + x^7)\, dx$

$= 3\left(\frac{x^{6/5}}{6/5}\right) - e\left(\frac{x^2}{2}\right) + \frac{x^8}{8} + C$

$\frac{3 \cdot 5}{6}$

$= \boxed{\frac{15}{6} x^{6/5} - \frac{e}{2} x^2 + \frac{1}{8} x^8 + C}$

EXTRA PROBLEM: $\int \left(\frac{2}{x} + \frac{3}{x^2}\right) dx = \int \left(\boxed{2} \cdot \frac{1}{x} + \boxed{3} x^{-2}\right) dx$

$= 2 \ln|x| + 3\left(\frac{x^{-1}}{-1}\right) + C = \boxed{2\ln|x| - 3x^{-1} + C}$

(f) $\int [6t(t+2)]\,dt = \int (6t^2 + 12t^1)\,dt$

$= 6\left(\frac{t^3}{3}\right) + 12\left(\frac{t^2}{2}\right) + C$

$= \boxed{2t^3 + 6t^2 + c}$

(g) $\int \left(\frac{5z^4 + 6z - 2}{z^3}\right) dz = \int \left(\frac{5z^4}{z^3} + \frac{6z}{z^3} - \frac{2}{z^3}\right) dz$

$\frac{z^4}{z^3} = \frac{\cancel{z}\cancel{z}\cancel{z}zz}{\cancel{z}\cancel{z}\cancel{z}}$

$\frac{z}{z^3} = \frac{\cancel{z}\,1}{\cancel{z}zz}$

$= \int (5z^1 + 6z^{-2} - 2z^{-3})\,dz$

$= 5\left(\frac{z^2}{2}\right) + 6\left(\frac{z^{-1}}{-1}\right) - 2\left(\frac{z^{-2}}{-2}\right) + C$

$= \boxed{\frac{5}{2}z^2 - 6z^{-1} + z^{-2} + c}$

(h) $\int (3x^2\sqrt{x^3+6})\,dx = \int 3x^2 (x^3+6)^{1/2}\, dx$

$\int u^{1/2}\, du$

$u = x^3+6$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

$= \frac{u^{3/2}}{3/2} + C = \boxed{\frac{2}{3}(x^3+6)^{3/2} + C}$

(i) $\int (8ye^{-2y^2+3})\,dy = \int 8y\, e^{-2y^2+3}\, dy$

$\int e^{u}(-2du)$

$u = -2y^2+3$

$\frac{du}{dy} = -4y$

$(-2)\,du = (-4y\,dy)(-2)$

$-2du = 8y\,dy$

$\int -2e^{u}\, du$

$-2e^{u} + C = \boxed{-2e^{-2y^2+3} + C}$

(j) $\int \left(\frac{x-2}{2x^2-8x+6} \right) dx$

$u = 2x^2 - 8x + 6$

$\frac{du}{4} = \frac{(4x-8)\,dx}{4} = \frac{4(x-2)\,dx}{4}$

$\frac{1}{4}du = (x-2)\,dx$

$\int \frac{\frac{1}{4}du}{u}$

$\int \frac{1}{4}\left(\frac{1}{u}\right) du = \frac{1}{4}\ln|u| + C = \boxed{\frac{1}{4}\ln\left|2x^2-8x+6\right| + C}$

(k) $\int t(t-8)^{100}\, dt$

$u = t - 8 \longrightarrow u + 8 = t$

$du = 1\, dt$

$\int \underbrace{(u+8)u^{100}}_{\text{multiply together}} du = \int \left(u^{101} + 8u^{100}\right) du$

$= \frac{u^{102}}{102} + 8\left(\frac{u^{101}}{101}\right) + C = \boxed{\frac{1}{102}(t-8)^{102} + \frac{8}{101}(t-8)^{101} + C}$

(l) $\int \left(\frac{1}{x \ln x}\right) dx$

$u = \ln x$

$du = \left(\frac{1}{x}\right) dx$

$\int \left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right) dx$

$\int \frac{1}{u} du$

$= \ln |u| + C = \boxed{\ln |\ln x| + C}$

(m) $\int \left(\frac{e^{2y}}{(e^{2y}+3)^4}\right) dy$

$u = e^{2y} + 3$

$\frac{du}{2} = \frac{2e^{2y}\, dy}{2}$

$\frac{1}{2} du = e^{2y} dy$

$(e^{2y})' = e^{2y}(2)$

$\int \frac{\frac{1}{2} du}{u^4}$

$\int \frac{1}{2} u^{-4} du = \frac{1}{2}\left(\frac{u^{-3}}{-3}\right) + C = \boxed{\frac{-1}{6}(e^{2y}+3)^{-3} + C}$

2. Find $f(x)$ if $f'(x) = \dfrac{2-x^2}{x^4}$ and $\boxed{f(1) = 2.}$ use this info to solve for C

$$f(x) = \int f'(x)\,dx$$

$$= \int \left(\frac{2-x^2}{x^4} \right) dx$$

$$= \int \left(\frac{2}{x^4} - \frac{x^2}{x^4} \right) dx$$

$$= \int \left(2x^{-4} - x^{-2} \right) dx$$

$$= 2\left(\frac{x^{-3}}{-3} \right) - \left(\frac{x^{-1}}{-1} \right) + C$$

$$f = \frac{-2}{3}\left(\frac{1}{x^3} \right) + \left(\frac{1}{x} \right) + C$$

$$f(1) = \frac{-2}{3}\left(\frac{1}{1^3} \right) + \left(\frac{1}{1} \right) + C = 2$$

$$-\frac{2}{3} + 1 + C = 2$$

$$\frac{1}{3} + C = 2$$

$$C = 2 - \frac{1}{3} = \frac{5}{3}$$

$$\boxed{f(x) = -\frac{2}{3}\left(\frac{1}{x^3} \right) + \frac{1}{x} + \frac{5}{3}}$$

3. Find y if $\underbrace{\frac{dy}{dx}}_{y'} = 500 - 0.5e^x$ and $\boxed{y(0) = 2.}$

$$y = \int y'\,dx$$

$$= \int (500 - 0.5e^x)\,dx$$

$$y = 500x - 0.5e^x + C$$

$$y(0) = 500(0) - 0.5e^0 + C = 2$$

$$0 - 0.5(1) + c = 2$$

$$-0.5 + c = 2$$

$$c = 2 + 0.5 = 2.5$$

$$\boxed{y = 500x - 0.5e^x + 2.5}$$

4. The monthly marginal revenue function for KNB Co. is given by $MR(x) = 10 - 0.01x + \dfrac{150}{x+2}$ where x is the number of thousands of items produced and sold and $MR(x)$ is measured in thousands of dollars per thousands of items. Find the revenue function for KNB Co.

$$MR(x) = R'(x) = 10 - 0.01x + \frac{150}{x+2}$$

$$Rev = R(x) = \int \left(10 - 0.01x + \frac{150}{x+2}\right) dx$$

$$= \int 10\,dx - \int 0.01x\,dx + \int \frac{150}{x+2}\,dx$$

$$= 10x - 0.01\left(\frac{x^2}{2}\right) + \boxed{\int \frac{150}{x+2}\,dx}$$

$$u = x+2$$

$$150\,du = (1\,dx)\,150$$

$$\int \frac{150}{x+2}\,dx = \int \frac{150}{u}\,du$$

$$= \int 150\left(\frac{1}{u}\right)du$$

$$= 150 \ln|u|$$

$$= \boxed{150 \ln|x+2|}$$

$$R = 10x - 0.005x^2 + 150 \ln|x+2| + C$$

$$* R(0) = 0 *$$

$$R(0) = 10(0) - 0.005(0)^2 + 150 \ln|0+2| + C = 0$$

$$0 - 0 + 150 \ln 2 + C = 0 \longrightarrow C = -150 \ln 2$$

$$\Rightarrow \boxed{R(x) = 10x - 0.005x^2 + 150 \ln|x+2| - 150 \ln 2}$$

5. A certain weed grows at the rate of $0.5e^{0.1t}$ cm/day. If the weed is 6 cm tall after 2 days, how tall is the weed after t days?

H t

Height of weed $= \int 0.5 e^{\boxed{0.1t}} dt$

$u = 0.1t$

$5\,du = (0.1\,dt)5$

$\underbrace{5\,du} = 0.5\,dt$

$\int e^{u}(5\,du)$

$\int 5e^{u}\,du$

$= 5e^{\boxed{u}} + C$

Height $= 5e^{0.1t} + C$

$6 = 5e^{0.1(2)} + C$

$6 - 5e^{0.2} = C$

$-0.107 \approx C$

$\boxed{\text{Height} \cong 5e^{0.1t} - 0.107}$

6. The rate of change of sales of a brand new soup (in thousands per month) is given by $R(t) = \sqrt{t} + 2$, where t is the time in months that the new product has been on the market. Find the total sales after 9 months.

$$\text{Total Sales} = \int (\sqrt{t} + 2)\,dt = \int (t^{1/2} + 2)\,dt$$

$$= \frac{t^{3/2}}{3/2} + 2t + C$$

$$\text{Total Sales} = \frac{2}{3} \cdot t^{3/2} + 2t + C$$

At $t=0$, since new soup, no sales!

$$0 = \frac{2}{3}(0)^{3/2} + 2(0) + C$$

$$0 = 0 + 0 + C \longrightarrow C = 0$$

$$\Rightarrow \text{Sales} = \frac{2}{3}t^{3/2} + 2t$$

After 9 mo, sales $= \frac{2}{3}(9)^{3/2} + 2(9)$

$$= \frac{2}{\cancel{3}}(\cancel{27}^{9}) + 18 = 18 + 18 = 36 \Rightarrow \boxed{36{,}000 \text{ cans}}$$

7. Approximate the area under the curve $y = \underbrace{x^2 + 2}$ and above the x-axis from $\underbrace{x = 2}_{a}$ to $\underbrace{x = 4}_{b}$, with $\underbrace{4}_{n}$ rectangles, using

(a) $\boxed{\text{left}}$ hand sums.

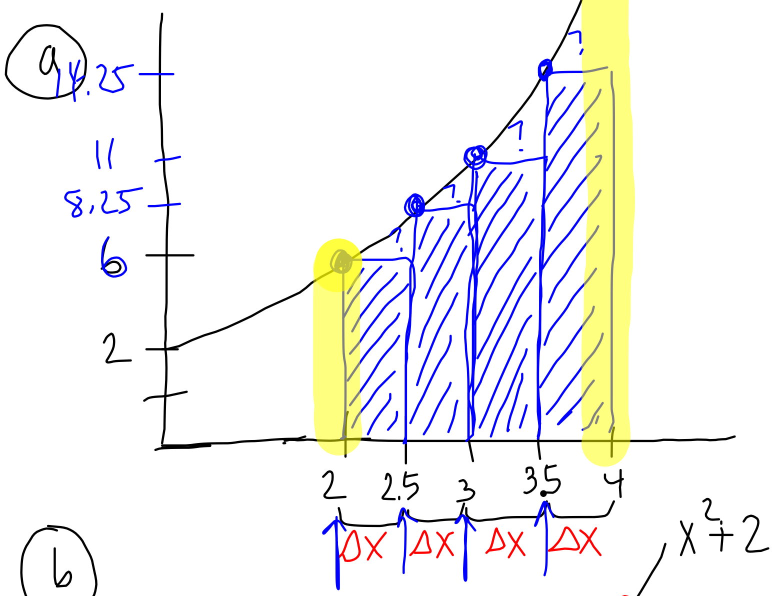
(b) $\boxed{\text{right}}$ hand sums.

width of rect $= \Delta x = \dfrac{b-a}{n}$

$= \dfrac{4-2}{4} = \dfrac{1}{2}$

x^2+2

(a)



x	Height x^2+2
2	6
2.5	$(2.5)^2+2 = 8.25$
3	11
3.5	$(3.5)^2+2 = 14.25$

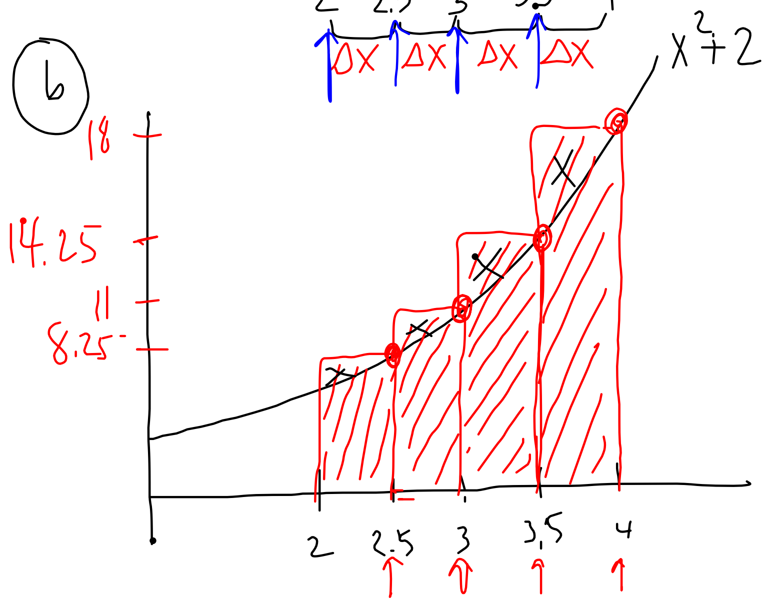
$LHS = L_{4} = \frac{1}{2}(6) + \frac{1}{2}(8.25) + \frac{1}{2}(11) + \frac{1}{2}(14.25)$

$= \frac{1}{2}\left[6 + 8.25 + 11 + 14.25\right]$

$= 19.75$

underestimate of true area

(b)



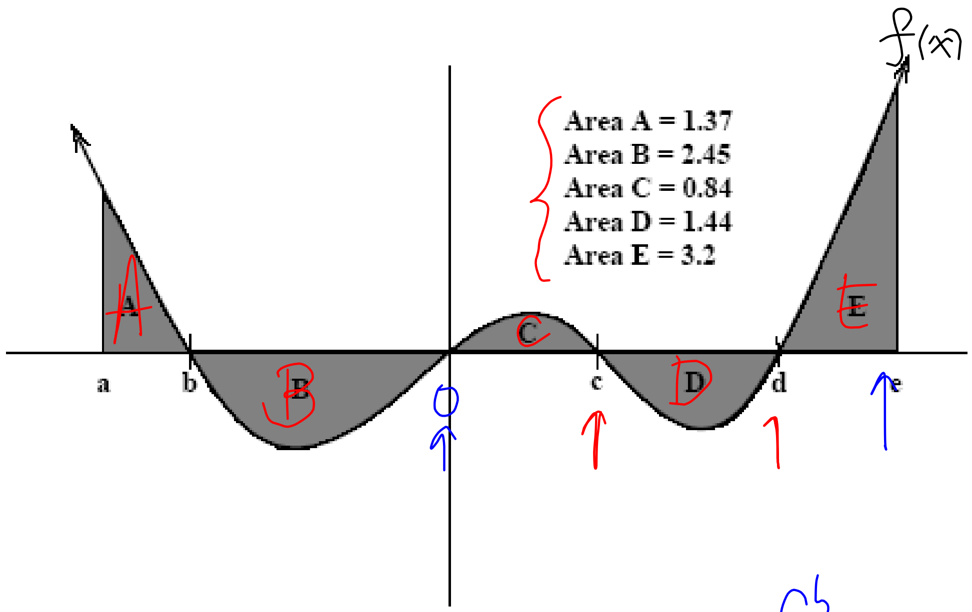
x	x^2+2
2.5	8.25
3	11
3.5	14.25
4	18

$RHS = R_{4}$

$= \frac{1}{2}\left[8.25 + 11 + 14.25 + 18\right]$

$= 25.75$ overestimate of true area

8. Using the given graph of $f(x)$, calculate the following:



(a) $\int_a^b f(x)\,dx = \boxed{1.37}$

(b) $\int_b^0 f(x)\,dx \; \boxed{-2.45}$

(c) $\int_a^0 f(x)\,dx = 1.37 - 2.45 = \boxed{-1.08}$

(d) $\int_0^e f(x)\,dx = 0.84 - 1.44 + 3.2$
$= \boxed{2.6}$

Definite Integral $\int_a^b f(x)dx$ is the NET area bt. $f(x)$ and the x-axis from $x=a$ to $x=b$.

EXTRA PROBLEM $\int_d^c f(x)\,dx = ?$

$= -\int_c^d f(x)dx = -1[-1.44] = \boxed{1.44}$

9. Given $\int_0^4 x\,dx = 8$, $\int_0^4 x^2\,dx = 64/3$ and $\int_4^6 x^2\,dx = 152/3$, calculate the following:

(a) $\int_0^4 2x^2\,dx$ $= 2 \boxed{\int_0^4 x^2\,dx} = 2\left(\frac{64}{3}\right) = \boxed{\frac{128}{3}}$

(b) $\int_0^6 (\boxed{-4}x^2)\,dx$ $= -4 \int_0^6 x^2\,dx = -4\left[\boxed{\int_0^4 x^2\,dx} + \boxed{\int_4^6 x^2\,dx}\right]$

$= -4\left[\frac{64}{3} + \frac{152}{3}\right] = -4(72)$

$= \boxed{-288}$

(c) $\int_0^4 (x^2 - 7x)\,dx$ $= \boxed{\int_0^4 x^2\,dx} - \int_0^4 \boxed{7}x\,dx$

$= \boxed{\int_0^4 x^2\,dx} - 7\boxed{\int_0^4 x\,dx} = \frac{64}{3} - 7(8) = \boxed{\frac{-104}{3}}$