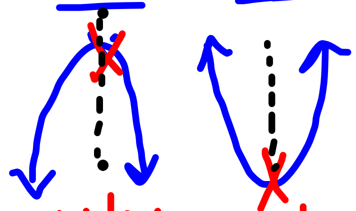
Week-In-Review #2 (1.4, 2.1)

Quad. Function

$f(x) = ax^2 + bx + c$

$a \neq 0$ $\quad a < 0 \quad a > 0$



vertex is a max — vertex is a min

axis of symm.

For each of the following quadratic functions,

(a) find its vertex (h,k) (is it a max or min?). $h = -b/2a$, $k = f(-b/2a)$

(b) find the intervals where the function is increasing/decreasing. $\underbrace{\text{intervals where}}_{x\text{-values}}$

(c) find all real roots of the function. zeros, x-int's $\Rightarrow y = 0$

(d) find the y-intercept. $(x=0)$

(e) draw a sketch of the function.

1. $f(x) = x^2 - 25$ $\quad a = 1$ (parabola opens ↑) $\quad b = 0 \quad c = -25$

(a) $x = \frac{-b}{2a} = \frac{-0}{2(1)} = 0$

$y = f(0) = 0^2 - 25 = -25$

vertex: $(0, -25)$ a minimum

(b) Dec: $(-\infty, 0)$

Inc: $(0, \infty)$

(c) $x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$x + 5 = 0 \qquad x - 5 = 0$

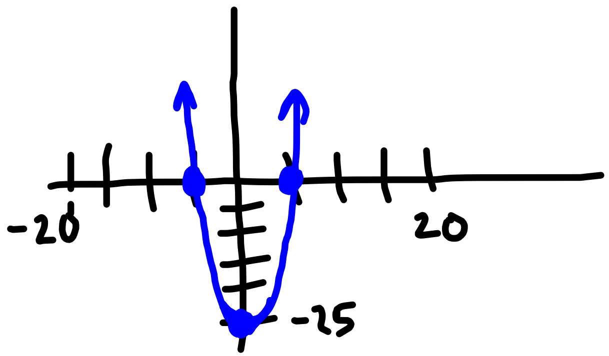
$x = -5 \qquad x = 5$

Roots at $x = \pm 5$

$(5,0)$ $(-5,0)$

(d) $f(0) = 0^2 - 25 = -25 \rightarrow$ y-int $(0, -25)$

(e)



2. $g(x) = 4x^2 - 8x + 3$ → opens ↑

"Complete the square": $g(x) = 4(x^2 - 2x + 1) + 3 - 4 = 4(x-1)^2 - 1$

$\left(-\frac{2}{2}\right)^2$

Vertex form:

$y = a(x-h)^2 + k$

vertex: (h, k)

(d) $g(0) = 4(0)^2 - 8(0) + 3 = 3$

y-int: $(0, 3)$

(a) vertex: $(1, -1)$ → a min

(b) Dec: $(-\infty, 1)$

Inc: $(1, \infty)$

(c) $4x^2 - 8x + 3 = 0$

$(2x - 1)(2x - 3) = 0$

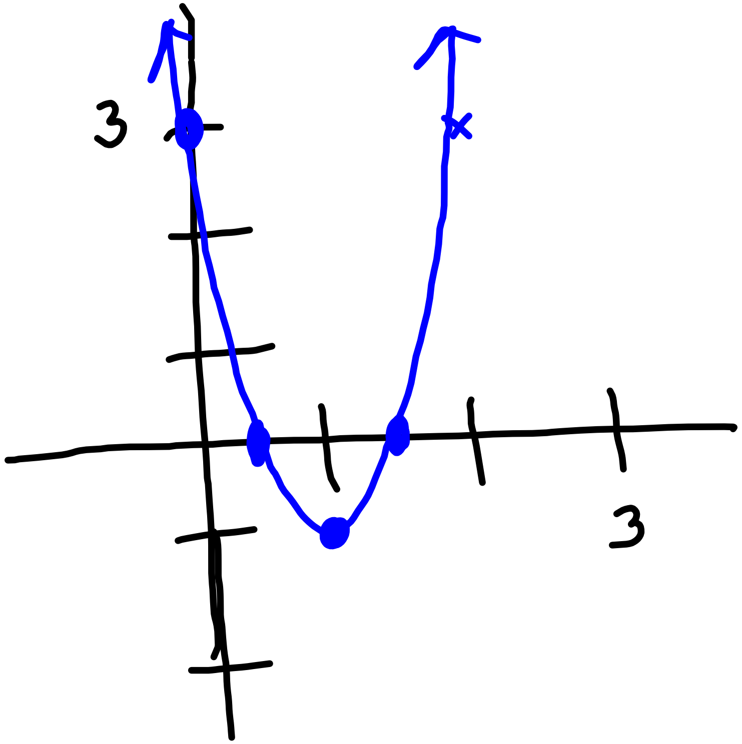
$2x - 1 = 0 \qquad 2x - 3 = 0$

$2x = 1 \qquad 2x = 3$

$x = 1/2 \qquad x = 3/2$

Roots: $\left(\frac{1}{2}, 0\right)$ $\left(\frac{3}{2}, 0\right)$

(e)



3. $h(x) = 1x^2 + 2x + 5$ $= (x^2 + 2x + 1) + 5 - 1 = (x+1)^2 + 4$

ⓐ vertex: $x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$y = h(-1) = (-1)^2 + 2(-1) + 5 = 1 - 2 + 5 = 4$ $\Big\}$ $(-1, 4)$ a min.

ⓑ <u>Dec</u>: $(-\infty, -1)$
<u>Inc</u>: $(-1, \infty)$

ⓒ $x^2 + 2x + 5 = 0 \rightarrow a = 1, b = 2, c = 5$

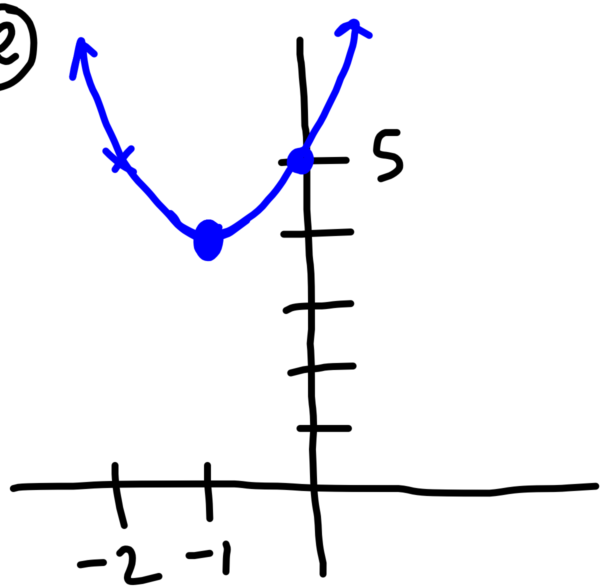
QUADRATIC FORMULA: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$

$= \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$ no real roots

ⓓ $h(0) = 0^2 + 2(0) + 5 = 5$
<u>y-int: $(0, 5)$</u>

ⓔ



4. $k(x) = -2x^2 + 3x + 10$ → opens ↓

(a) vertex:

$$x = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4} = 0.75$$

$$y = k\left(\frac{3}{4}\right) = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 10 = \frac{89}{8} = 11.125$$

$(0.75, 11.125)$ a max

(b) Inc: $(-\infty, 0.75)$

Dec: $(0.75, \infty)$

(c) $-2x^2 + 3x + 10 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-2)(10)}}{2(-2)}$$

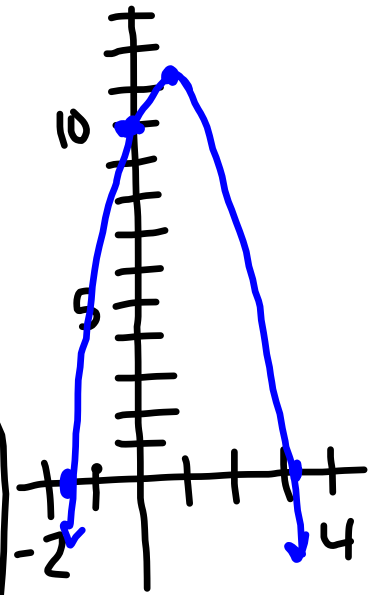
$$= \frac{-3 \pm \sqrt{9 + 80}}{-4}$$

$$= \frac{-3 \pm \sqrt{89}}{-4} \rightarrow x = \frac{-3 + \sqrt{89}}{-4} \quad x = \frac{-3 - \sqrt{89}}{-4}$$

$\approx -1.6085, \ 3.1085$

(d) $k(0) = -2(0)^2 + 3(0) + 10 = 10$

y-int: $(0, 10)$



*5. It is known that 1,500 (15) widgets can be sold when each widget sells for \$10 and 3,000 (30) can be sold when each sells for \$5. Additionally, it costs the company producing the widgets a total of \$300 to produce 10,000 (100) widgets and \$50 without producing any widgets. Let x = the number of widgets made and sold (in hundreds). Find the company's

(a) linear price-demand function.

$p = mx + b$ $\quad (15, 10)^*$ $\quad (30, 5)$

$* (x, p) *$

$m = \frac{\Delta p}{\Delta x} = \frac{5-10}{30-15} = \frac{-5}{15} = -\frac{1}{3}$ *

Pt-Slope Form: $y - y_1 = m(x - x_1)$

$p - 10 = -\frac{1}{3}(x - 15)$

$p - 10 = -\frac{1}{3}x + 5$

$\boxed{p = -\frac{1}{3}x + 15}$

(b) linear cost function.

$C(x) = cx + F$ (fixed costs)

$(100, 300)$

$*(0, 50)*$

$m = c = \frac{50-300}{0-100} = \frac{-250}{-100} = 2.5$

$\boxed{C(x) = 2.5x + 50}$

(c) revenue function.

$R = (\text{price})(\text{quantity}) = (\text{price-demand})(\text{quantity})$

$= \left(-\frac{1}{3}x + 15\right)(x) \longrightarrow \boxed{R = -\frac{1}{3}x^2 + 15x}$

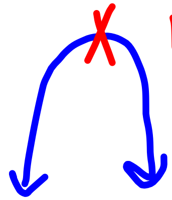
(d) profit function.

$P = R - C = \left(-\frac{1}{3}x^2 + 15x\right) - (2.5x + 50)$

$= -\frac{1}{3}x^2 + 15x - 2.5x - 50 \longrightarrow \boxed{P = -\frac{1}{3}x^2 + 12.5x - 50}$

(e) selling price for widgets in order to maximize their profits. What is the maximum profit?

$$P = -\frac{1}{3}x^2 + 12.5x - 50$$



max @ the vertex

Vertex:

$$x = \frac{-b}{2a} = \frac{-12.5}{2(-1/3)} = 18.75$$

$$y = P(18.75) = -\frac{1}{3}(18.75)^2 + 12.5(18.75) - 50 = 67.1875$$

$$(\underbrace{18.75}_{\text{\# of widgets to max profit}}, \underbrace{67.1875}_{\text{max profit}})$$

$$\boxed{\text{Max Profit: \$67.18}}$$

Selling price → use price-demand:

$$p = -\frac{1}{3}x + 15$$

$$p = -\frac{1}{3}(18.75) + 15$$

$$= 8.75$$

$$\boxed{\text{price} = \$8.75}$$

2.1

Polynomials: $f = a_n x^{\boxed{n}} + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

where the a's are real #s ($a_n \neq 0$)

* n = highest pwr of x
= degree of the poly.
(NON-NEGATIVE INTEGER)

a_n = leading coefficient

max # of $\underbrace{\text{turning points}}_{\text{changes bt. inc \& dec.}} = n-1$

max # of x-int $= n$

6. Which of the following are polynomials? If the function is a polynomial, state its degree, leading coefficient, maximum number of turning points, maximum number of possible x-intercepts, and the smallest interval where all of the x-intercepts can be found. poly. root approx.

(a) $f(x) = 7x^3 - ex^2 + \pi$

degree = 3
lead. coeff = 7
max # turning pts = 2
max # x-ints = 3

$\frac{1}{x^m} = x^{-m}$

$\frac{1}{x^2} = x^{-2}$

$\frac{1}{x^{-2}} = x^{-(-2)} = x^2$

(b) $g(x) = \sqrt{8x^5}$ $= (8x^5)^{1/2} = (8^{1/2})(x^5)^{1/2}$

$= (\sqrt{8})x^{\boxed{5/2}}$ not an integer → not a poly

(c) $h(x) = 4 - x^2 - \dfrac{3}{x^{-4}}$

$h = 4 - x^2 - 3x^4$

degree = 4
lead coeff = -3
max # of turn. pts = 3
max # of x-int = 4

(d) $m(x) = 1 + x^3 - 2x^5 + x^2$

degree = 5

lead. coeff = -2

max # of t.p. = 4

max # of x-int = 5

(e) $n(x) = (\sqrt{7})x^6 - 9$

degree = 6

lead. coeff. = $\sqrt{7}$

max # of t.pts = 5

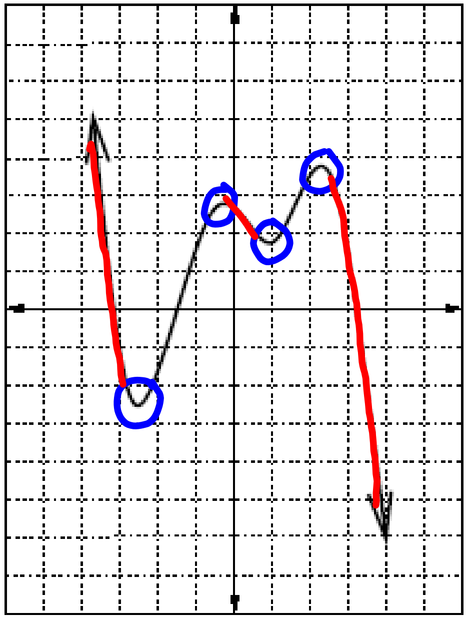
max # of x-int = 6

7. For the graph below,

(a) How many turning points are on the graph? 4

(b) What is the minimum degree of a polynomial function that could have the graph? 5 (will be odd)

(c) Is the leading coefficient of the polynomial positive or negative?



$y = x^3$ $\cancel{x^2}$

$y = 2x$

Odd Degree

$y = 2x^2$

Even Degree

$a_n > 0$

pos. lead. coeff.

$y = -2x$

$y = -2x^2$

$a_n < 0$

neg. lead. coeff.

8. The following table gives the population (in hundreds of people) of Podunkville, USA for some selected years.

	0	30	50	75	100	125	150	
Year(x)	1800	1830	1850	1875	1900	1925	1950	L_1
Population(y)	25	60	125	350	680	1400	2500	L_2

pop. grows

(a) Let $x = 0$ represent the year 1800. Find the best-fitting quadratic, cubic, and quartic models to this data. Round each coefficient to four decimal places, if necessary, in each model. Which of these do you think most accurately models the given data?

worst

QuadReg (Y_1) $(R^2 = .9884)$: $y = 0.1665x^2 - 9.7287x + \overbrace{111.5360}^{\text{init. pop in 1800}}$

high pop → neg. pop → high pop

* CubicReg $(R^2 = .9995)$: $y = 0.001x^3 - 0.0598x^2 + 2.7048x + \underbrace{19.4880}_{\text{init. pop. in 1800}}$

low pop --→ pop grows

QuartReg $(R^2 = 0.9997)$: $y = 0.000003x^4 + 0.00003x^3 + 0.0309x^2 + 0.1524x + \underbrace{24.3674}_{\text{1800 pop.}}$

higher pop → dec (not neg) → inc

(b) Use your unrounded model to predict the population in 1999.

$\underbrace{\text{predict the population}}_{y=?}$ $\underbrace{\text{in 1999}}_{x=199}$

Use eqn stored in calculator

$y_2 =$ cubicreg eqn

$y_2(199) = 6168.63869 \times 100 = 616863.869$

$\boxed{616,864 \text{ people}}$

(c) Use your unrounded model to estimate when Podunkville had 9600 people.

$\underbrace{\text{when}}_{x=?}$ $\underbrace{\text{9600 people}}_{y=96}$

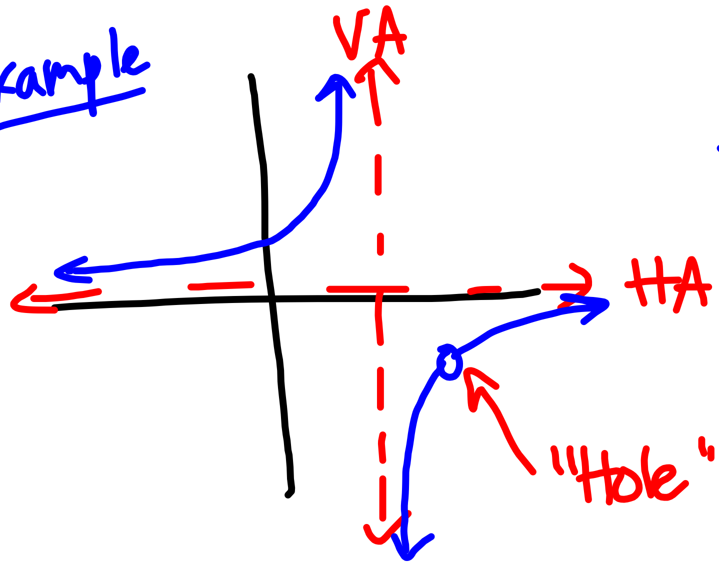
$y_2 =$ cubic reg. eqn

$y_1 = 96$

Calc - intersect $\rightarrow x = 39.72\ldots \Rightarrow \boxed{1839}$

Rational Function : $f = \frac{n(x)}{d(x)}$

example



$n(x), d(x)$ are polynomials

VA (vert. asymptote) : "x-value where y-values go to $\pm\infty$"

↳ when denom $= 0$ AND at the same time the num $\neq 0$

"Hole" → when an x-value makes BOTH the denom & num $= 0$

HA: What happens @ ends of the function:

1. If deg $n(x) >$ deg $d(x)$ → No HA (ends go to $+$ or $-\infty$)
2. If deg $n(x) <$ deg $d(x)$ → HA: $y = 0$
3. If degrees equal → HA: $y = a/b$ (a, b lead. coeff)

9. Find the domain and identify all asymptotes and "holes" in the graphs of the following functions:

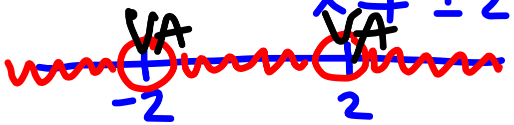
VA/HA

(a) $f(x) = \dfrac{x^2 - 1}{x^2 - 4} = \dfrac{(x+1)(x-1)}{(x+2)(x-2)}$

$1 \quad 1$

Domain: denom $\neq 0$

$x \neq \pm 2$



$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

VA: $x = 2$

$x = -2$

Holes: none

HA: Deg of num $= 2$] equal

Deg. of denom $= 2$

$y = \frac{1}{1} \rightarrow y = 1$

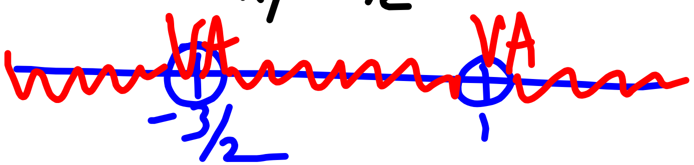
(b) $g(x) = \dfrac{-x + 5}{2x^2 + x - 3} = \dfrac{-x+5}{(2x+3)(x-1)}$

Domain: $(2x+3)(x-1) = 0$

$2x + 3 = 0 \qquad x - 1 = 0$

$2x = -3 \qquad x \neq 1$

$x \neq -3/2$



$(-\infty, -3/2) \cup (-3/2, 1) \cup (1, \infty)$

VA: $x = -3/2$, $x = 1$

Holes: none

HA: Higher deg. poly in denom

$\Longrightarrow$ HA: $y = 0$

(c) $h(x) = \dfrac{(x+4)\cancel{(x-2)}(4x+7)}{\cancel{(x-2)}(x+3)} = \dfrac{\text{cubic}}{\text{quadratic}}$

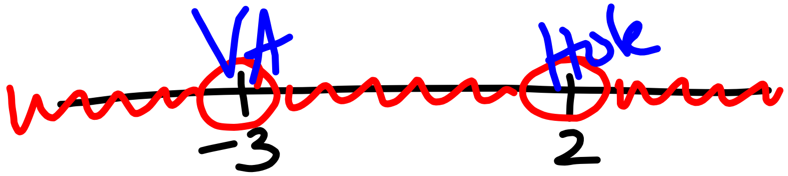
VA: $x = -3$

Holes: at $x = 2$

Domain: $(x-2)(x+3) = 0$

$x - 2 = 0 \quad x + 3 = 0$

$x \neq 2 \quad x \neq -3$



$(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

HA: higher deg. in numerator

$\Longrightarrow$ $\boxed{\text{NO HA}}$

Domain

10. Find all asymptotes, intercepts and and any "holes" which occur in the graph of the following function and then make a sketch of the function.

$$f(x) = \frac{(-2x+5)(x-1)}{(x+2)(3x-4)} = \frac{-2x^2+2x+5x-5}{3x^2-4x+6x-8}$$

plug in x=0

x-intercepts: (y=0)

$$0 = \frac{(-2x+5)(x-1)}{(x+2)(3x-4)}$$

$$0 = (-2x+5)(x-1)$$

$-2x+5=0$ $\quad$ $x-1=0$

$-2x=-5$

$x=5/2$ $\quad$ $x=1$

$$\boxed{(5/2, 0) \quad (1, 0)}$$

y-int: (x=0) $f(0) = \frac{(5)(-1)}{(2)(-4)}$

$= \frac{-5}{-8}$

$= 5/8$

$$\boxed{(0, 5/8)}$$

Domain: $(x+2)(3x-4)=0$

$x+2=0$ $\quad$ $3x-4=0$

$x \neq -2$ $\quad$ $3x=4$

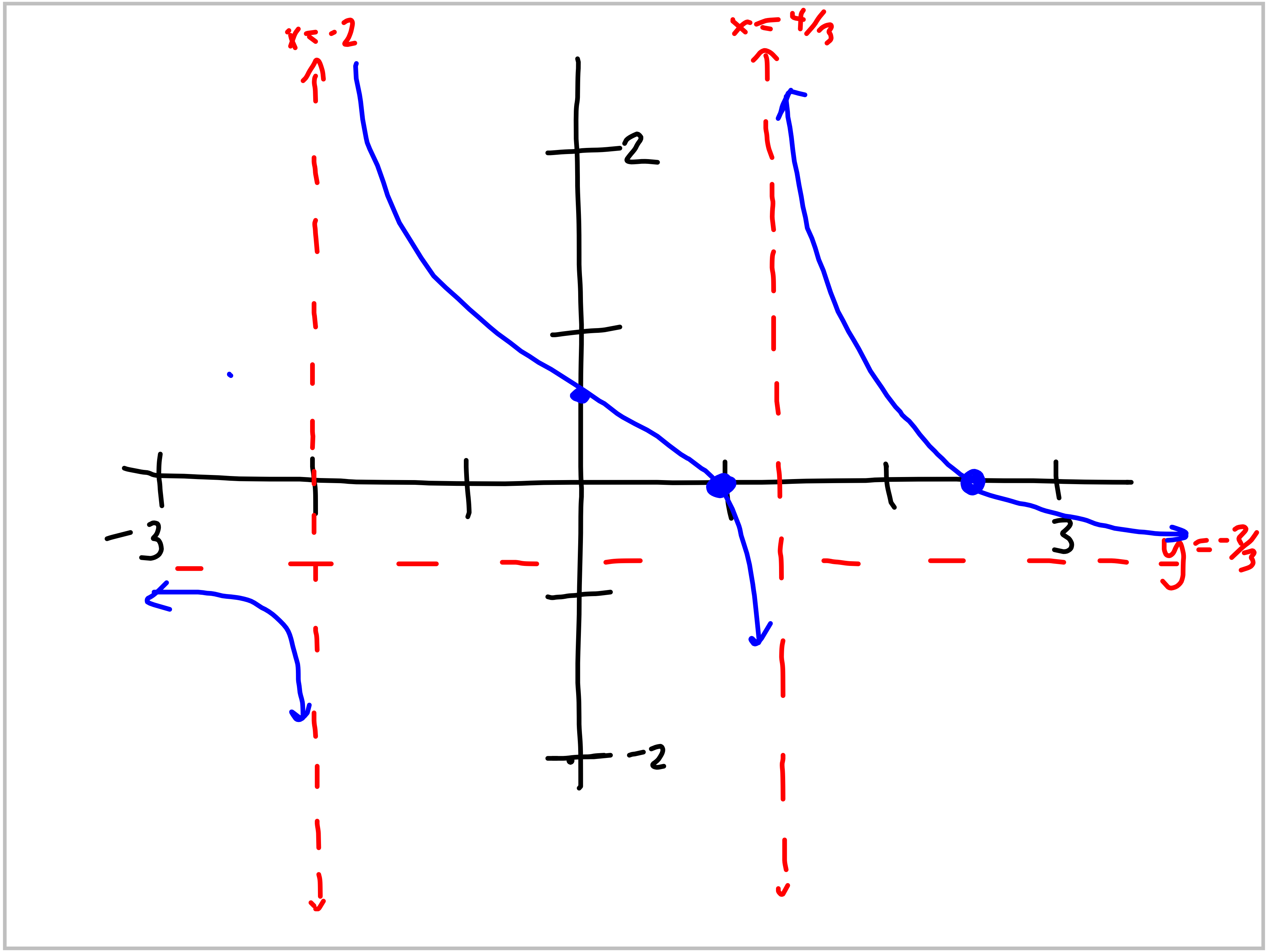
$x \neq 4/3$

VA: $x=-2$, $x=4/3$

no Holes

HA: Same degree

$$\boxed{y = \frac{-2}{3}}$$



x = -2
x = 4/3
2
-2
-3
3
y = -2/3