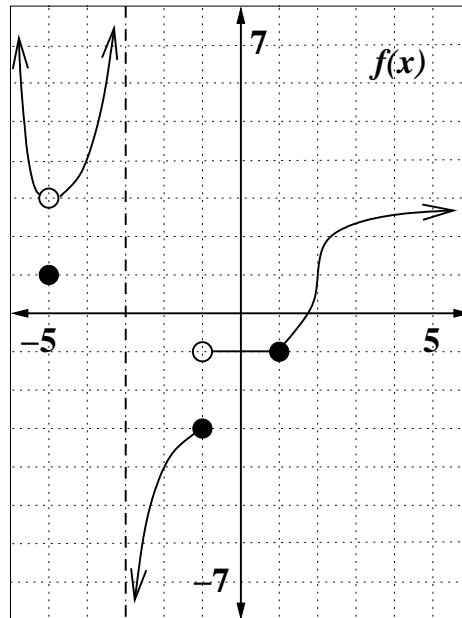


## Week-In-Review #5

1. Use the given graph of  $f(x)$  to answer the following questions.



(a)  $\lim_{x \rightarrow -1^-} f(x) =$

(b)  $\lim_{x \rightarrow -1^+} f(x) =$

(c)  $\lim_{x \rightarrow -1} f(x) =$

(d)  $\lim_{x \rightarrow -3^+} f(x) =$

(e)  $\lim_{x \rightarrow 0} f(x) =$

(f)  $\lim_{x \rightarrow 1^+} f(x) =$

(g)  $\lim_{x \rightarrow -5} f(x) =$

(h)  $\lim_{x \rightarrow \infty} f(x) =$

(i)  $\lim_{x \rightarrow -\infty} f(x) =$

Find the following limits:

$$2. \lim_{x \rightarrow 1} \frac{2x^2 + 5x - 4}{x + 2}$$

$$3. \lim_{x \rightarrow 5} \frac{x^2 - 3x - 10}{x - 5}$$

$$4. \lim_{h \rightarrow 0} \frac{2x + 3h}{x}$$

$$5. \lim_{x \rightarrow 0} \frac{x^4}{x^3 + 2}$$

$$6. \lim_{x \rightarrow 3} \frac{|x - 3|}{x - 3}$$

7. Given  $f(x) = \begin{cases} x + 3 & , x < -1 \\ 0 & , x = -1 \\ x^2 + 1 & , x > -1 \end{cases}$ , find

(a)  $\lim_{x \rightarrow -1^-} f(x)$

(b)  $\lim_{x \rightarrow -1^+} f(x)$

(c)  $\lim_{x \rightarrow -1} f(x)$

(d)  $\lim_{x \rightarrow -3} f(x)$

(e)  $\lim_{x \rightarrow 5} f(x)$

8.  $\lim_{x \rightarrow -1^+} \frac{x + 2}{x^2 + 3x + 2}$

9.  $\lim_{x \rightarrow 0^-} \frac{-5}{x^2}$

10.  $\lim_{x \rightarrow \infty} \frac{2x^2 + 4x - x^3}{3 + 7x^3 + 8x}$

11.  $\lim_{x \rightarrow -\infty} \frac{x^2 + 2x + 5}{2x^2 + 3x^4}$

12.  $\lim_{x \rightarrow -\infty} \frac{x^2 + 2x}{x + 5}$

$$13. \lim_{x \rightarrow \infty} \frac{2e^x + 3e^{-x}}{5e^{-x} - 7e^x}$$

$$14. \lim_{x \rightarrow -\infty} \frac{2e^x + 3e^{-x}}{5e^{-x} - 7e^x}$$

$$15. \text{ Given } f(x) = \begin{cases} \frac{2x^2 + 1}{x^2 + 1} & , x < 0 \\ \frac{x + 5}{x^3 + 8} & , x \geq 0 \end{cases} \text{ , find}$$

$$(a) \lim_{x \rightarrow \infty} f(x)$$

$$(b) \lim_{x \rightarrow -\infty} f(x)$$

16. Where are the following functions continuous?

$$(a) f(x) = 3x^7 + 8x^5 - 8x + 10$$

$$(b) f(x) = 5e^{(x-2)} + 17$$

$$(c) f(x) = \frac{x^2 - 4}{x^3 - 8}$$

$$(d) f(x) = 2 \ln(2x - 7) + 5$$

17. Where is the following function discontinuous?

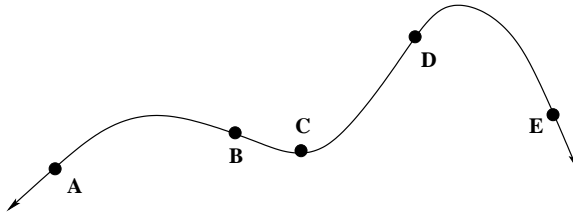
$$f(x) = \begin{cases} \frac{x+1}{x^2-9} & , x \leq 0 \\ 2x+4 & , x > 0 \end{cases}$$

18. Find the values of  $a$  and  $b$  so that  $g(x)$  is continuous on  $(-\infty, \infty)$ , if possible.

$$f(x) = \begin{cases} 2a+6x & , x < 1 \\ 4 & , x = 1 \\ x-b & , x > 1 \end{cases}$$

19.

- (a) Between which consecutive pairs of points in the following graph is the average rate of change positive? negative? zero?
- (b) Put the points labeled in the following graph in order (from smallest to largest) according to the value of the instantaneous rate of change of the function at that point.

20. Given  $f(x) = x^3 - x + 3$ , evaluate the following:

- (a)  $f(1)$
- (b)  $f(a)$
- (c)  $f(\star)$
- (d)  $f(\Delta)$
- (e)  $f(Kathryn)$
- (f)  $f(Kathryn + Bollinger)$
- (g)  $f(Math + x)$
- (h)  $f(x + h)$

21. Using the limit definition of derivative, find the derivatives of the following functions:

(a)  $f(x) = x^2 + 3x - 7$

(b)  $g(x) = \frac{x}{x-2}$

(c)  $k(x) = \sqrt{x+1}$

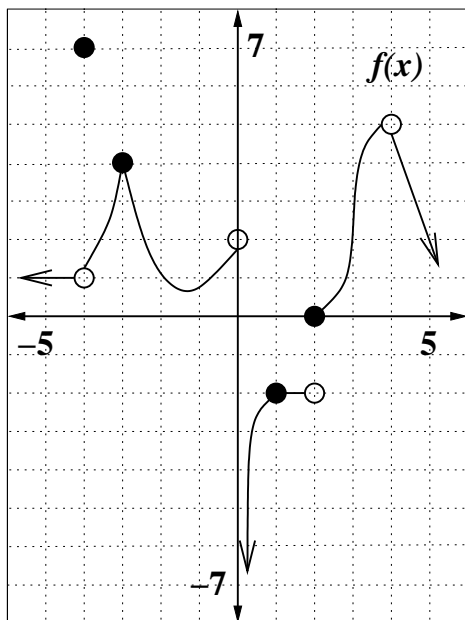
22. Given  $f(x) = 2x^2 + 4$ , find

(a) the average rate of change of  $f(x)$  when  $x$  changes from 0 to 2.

(b) the instantaneous rate of change of  $f(x)$  at  $x = 0$ .

(c) The equation of the tangent line to  $f(x)$  at  $x = 0$ .

23.



(a) Where is  $f(x)$  discontinuous? Justify your answer using the definition of continuity.

(b) Where is  $f(x)$  not differentiable? Justify your answer.

(c) Where is the instantaneous rate of change of  $f(x)$  zero?