Week-In-Review #7

First deriv. of $f = f'$
2nd deriv $= f''$
3rd " $= f'''$

1. Find the first three derivatives of the following functions:

(a) $f(x) = \boxed{(5x+2)}^2$

$f' = 2(5x+2)^1(5) = 10(5x+2)$
$= 50x + 20$

$f'' = 50$

$f''' = 0$

(b) $g(x) = \sqrt[5]{x^3} = x^{3/5}$

$g' = \frac{3}{5} x^{-2/5}$

$g'' = \frac{-6}{25} x^{-7/5}$

$g''' = \frac{42}{125} x^{-12/5}$

(c) $h(x) = (4x)(2x+6)^3)$

PROD RULE $h' = 4(2x+6)^3 + (4x)[3(2x+6)^2(2)]$

$= \underbrace{4(2x+6)^3} + (24x)(2x+6)^2)$

$h'' = 12(2x+6)^2(2) + \{24(2x+6)^2 + (24x)[2(2x+6)^1(2)]\}$

$= \underbrace{24(2x+6)^2 + 24(2x+6)^2} + 96x(2x+6)$

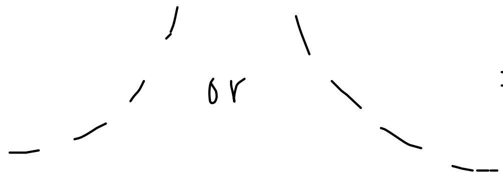
$= \underbrace{48(2x+6)^2} + \underbrace{(96x)(2x+6)}$

$h''' = 96(2x+6)^1(2) + \{96(2x+6) + (96x)(2)\}$

For each graph given in questions #2-#5, determine the intervals where

(a) $f'(x) > 0$ $\longrightarrow$ m_{tan} of f positive / $\Rightarrow$ f is increasing

(b) $f'(x) < 0$ $\longrightarrow$ m_{tan} of f negative \ $\Rightarrow$ f is decreasing

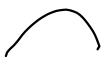
(c) $f''(x) > 0$ $\rightarrow$ m_{tan} of f are inc  $\Rightarrow$ f is concave up

++

f 

(d) $f''(x) < 0$ $\rightarrow$ m_{tan} of f are dec  $\Rightarrow$ f is concave down

− −

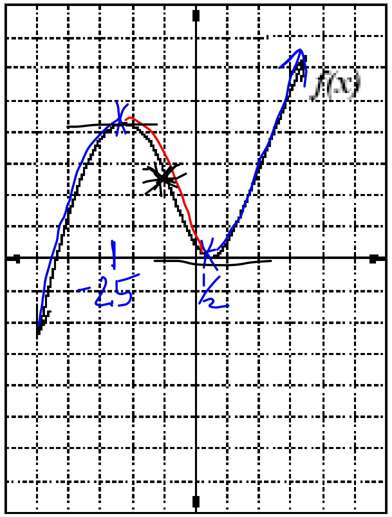
f 

and list any x-values where $\underbrace{f'(x) = 0}_{m_{tan} = 0}$ or where $f'(x)$ is undefined.

$m_{tan} = 0$
$\Rightarrow$ horiz. tangent lines

where $f(x)$ is discontinuous
$f(x)$ has sharp turns
$f(x)$ has a vert. tan. line

2.



(a) $(-\infty, -2.5)$ $(\frac{1}{2}, \infty)$

(b) $(-2.5, \frac{1}{2})$

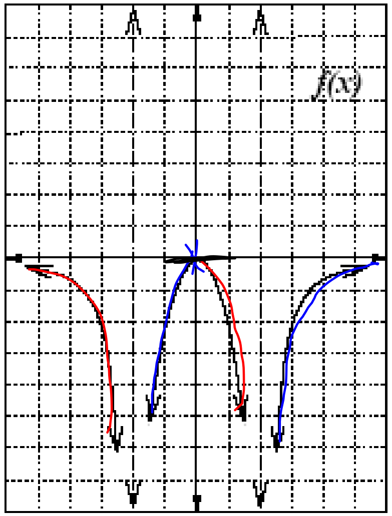
(c) $\cup$: $(-1, \infty)$

(d) $\cap$: $(-\infty, -1)$

(e) $f' = 0$: @ $x = -2.5$ &
$x = \frac{1}{2}$

f' DNE? never

3.



(a) $(-2, 0)$ $(2, \infty)$

(b) $(-\infty, -2)$ $(0, 2)$

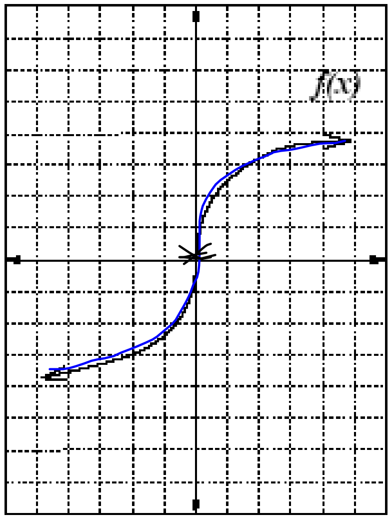
(c) $\cup$: never

(d) $\cap$: $(-\infty, -2)$ $(-2, 2)$
$(2, \infty)$

(e) $f' = 0$: @ $x = 0$

f' DNE? @ $x = -2$ & $x = 2$
(these values are not in the domain of f.)

4.



a $(-\infty, 0)$ $(0, \infty)$

b never

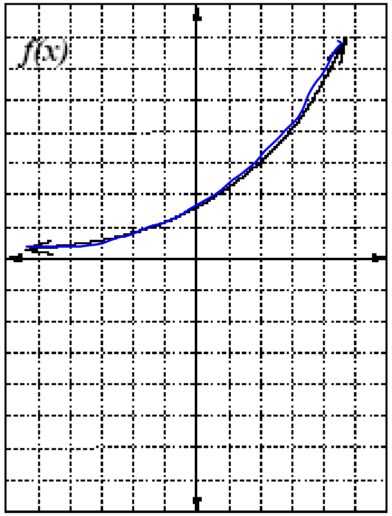
c $\cup$: $(-\infty, 0)$

d $\cap$: $(0, \infty)$

e $\underline{f' = 0}$? never

f' DNE @ $x = 0$
(vert. tan.)

5.



a $(-\infty, \infty)$

b never

c $\cup$: $(-\infty, \infty)$

d $\cap$: never

e $\underline{f' = 0}$? never

$\underline{f' \text{ DNE}}$? never

6. Given $f'(x) = n(x+2)(x-3)^4(x+5)^3$, where n is a function that is always negative, determine critical values of $f(x)$, intervals where $f(x)$ is increasing or decreasing and any values of x where local extrema of $f(x)$ will occur.

Local Max ↗ ↘   Local min ↘ ↗

* Since not told otherwise, the domain of $f(x)$ is assumed to be $\mathbb{R}$.

Critical values (CVs): x-values $\boxed{\text{in the domain of } f(x)}$ where $f'(x) = 0$ or $f'(x)$ DNE

$f' = 0$: $(n)(x+2)(x-3)^4(x+5)^3 = 0$

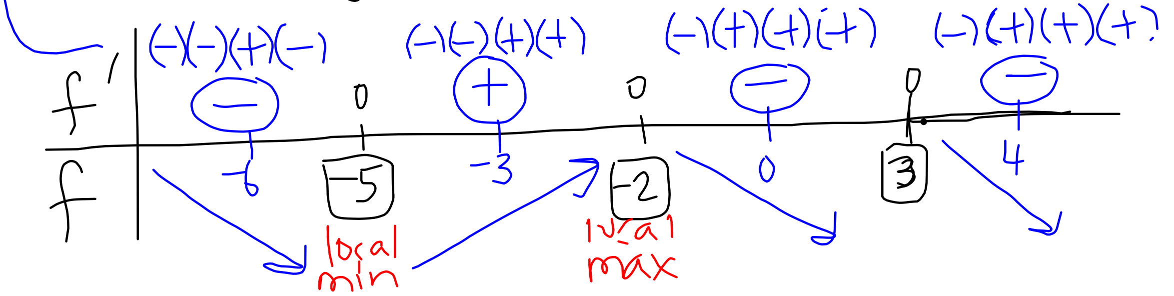
$n \neq 0$ $(x+2) = 0$ $(x-3)^4 = 0$ $(x+5)^3 = 0$

$\boxed{x = -2}$ $x - 3 = 0$ $x + 5 = 0$

$\boxed{x = 3}$ $\boxed{x = -5}$

n always neg. for every x-value

f' defined everywhere $\Longrightarrow$ $\boxed{\text{CVs: } x = -5, -2, 3}$



Inc: $(-5, -2)$

Dec: $(-\infty, -5)$ $(-2, 3)$ $(3, \infty)$

Local min @ $x = -5$

Local max @ $x = -2$

7. Determine the concavity and any inflection points of $f(x) = 4x - x^4$, using the second derivative.

$f'' > 0 \rightarrow f$ is concave up $++$

$f'' < 0 \rightarrow f$ is concave dwn $--$

Inflection pt: occurs at a point in the domain of f where concavity changes (f'' changes sign)
(These pts will also have $f''=0$ or f'' DNE.)

Domain of $f(x)$: $\mathbb{R}$

$f' = 4 - 4x^3$

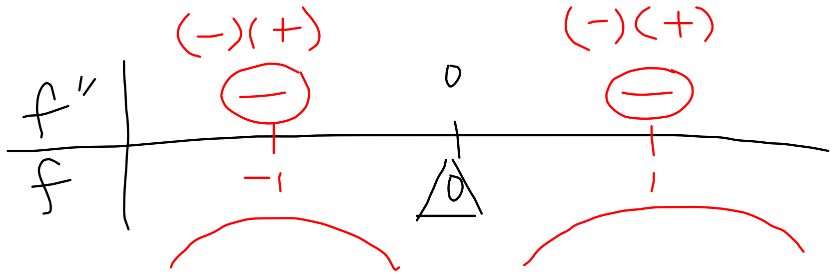
$f'' = -12x^2$

$f''=0$: $-12x^2 = 0$

$x^2 = 0$

$x = 0$

f'' def. everywhere



Concave ↓ : $(-\infty, 0)\ (0, \infty)$

Concave ↑ : never

No Inf. Pts.

For each function in questions #8-#10, find the following information and use it to sketch each function.

From $f(x)$

(a) Domain of $f(x)$ Poly: $\mathbb{R}$ Even Root: $\sqrt[n]{☆} \to ☆ \geq 0$ Rational: $\frac{T}{B} \to B \neq 0$

(b) Intercepts x-int $\to y=0$ $(?,0)$ y-int $\to x=0$ $(0,?)$

(c) Asymptotes/Holes

Holes: x-values that make both the numerator & denominator $= 0$.

VA: Simplify rational & then what makes only denom $= 0$.

HA: $\lim_{x \to \pm\infty} f(x)$ (w/ a rational function you can use highest pwrs rule)

From $f'(x)$

(d) All critical values

where $f'=0$ or f' DNE IN THE DOMAIN OF $f(x)$.

(e) All intervals where $f(x)$ is increasing/decreasing Inc: $f'>0$ Dec: $f'<0$

(f) Any local extrema (occur @ CVs)

Local MAX: $\nearrow \searrow$ $f'=0$ or f' DNE

Local MIN: $\searrow \nearrow$ $f'=0$ or f' DNE

From $f''(x)$

(g) All intervals where $f(x)$ is concave up/concave down CU: $f''>0$ CD: $f''<0$

(h) Any inflection points

Where $f''(x)$ changes sign (the point will be in the domain & $f''=0$ or f'' DNE)

I OR I

8. $f(x) = x^4 - 6x^2 + 5$

(a) Domain: $\mathbb{R}$

(b) x-int: $(y=0)$

$0 = x^4 - 6x^2 + 5$

$0 = (x^2 - 1)(x^2 - 5)$

$0 = x^2 - 1$ or $0 = x^2 - 5$

$x^2 = 1 \qquad x^2 = 5$

$x = \pm 1 \qquad x = \pm\sqrt{5}$

y-int: $(x=0)$: $f(0) = 0^4 - 6(0)^2 + 5 = 5$

(c) Since f is a poly, no Asymp/holes

(d) $f'(x) = 4x^3 - 12x$

$= 4x(x^2 - 3)$

$f' = 0 = (4x)(x^2 - 3)$

$4x = 0 \qquad x^2 - 3 = 0$

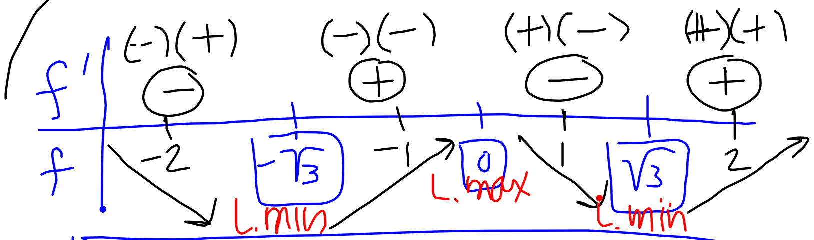
$x = 0 \qquad x^2 = 3$

$x = \pm\sqrt{3}$

$\boxed{\text{CVs: } x = -\sqrt{3}, 0, \sqrt{3}}$

f' def. everywhere

(e) $f' = (4x)(x^2 - 3)$



Inc: $(-\sqrt{3}, 0)\ (\sqrt{3}, \infty)$

Dec: $(-\infty, -\sqrt{3})\ (0, \sqrt{3})$

(f) next page ⟹

(g) $f' = 4x^3 - 12x$

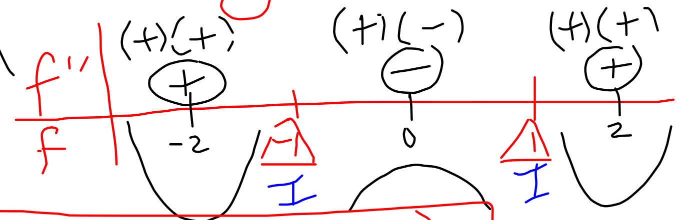
$f'' = 12x^2 - 12 = (12)(x^2 - 1)$

$f'' = 0 = (12)(x^2 - 1)$

$12 \neq 0 \qquad x^2 - 1 = 0$

$x^2 = 1 \rightarrow x = \pm 1$

f'' def. everywhere



CU: $(-\infty, -1)\ (1, \infty)$

CD: $(-1, 1)$

(h) next pg ⟹

IMPT PTS

x-int : $(-1,0)\ (1,0)\ (-\sqrt{5},0)\ (\sqrt{5},0)$

y-int : $(0,5)$

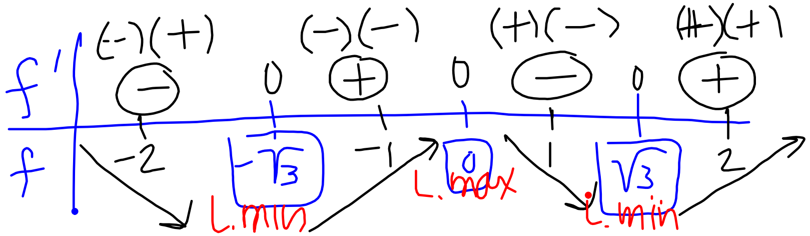
Local max: $(0,f(0)) = (0,5)$

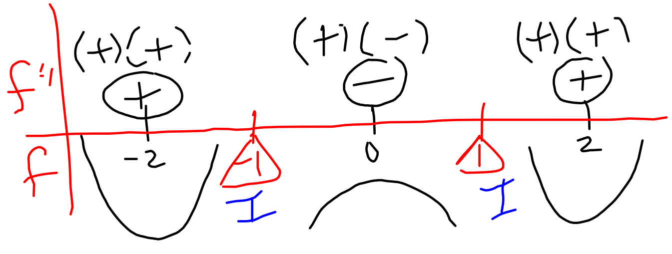
Local min : $(-\sqrt{3}, f(-\sqrt{3})) = (-\sqrt{3},-4)$

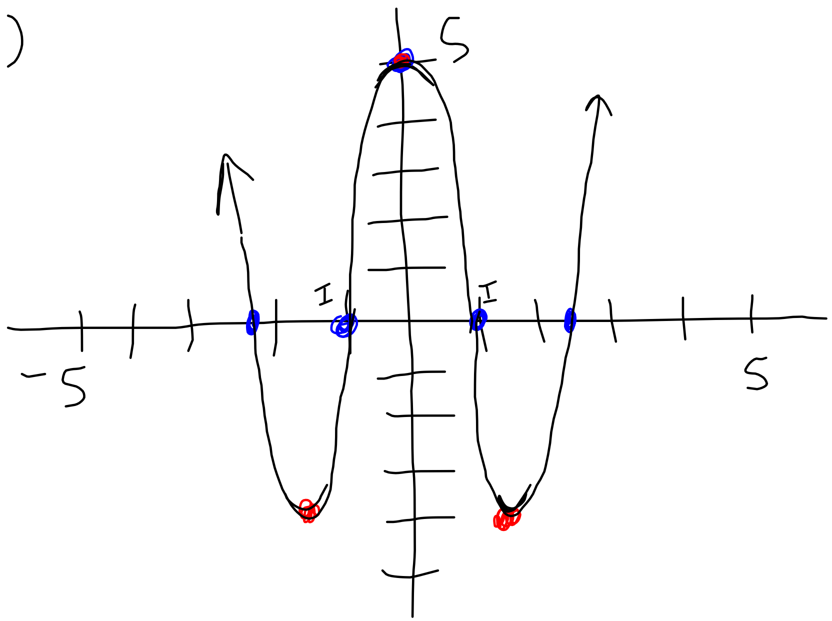
$(+\sqrt{3}, f(\sqrt{3})) = (+\sqrt{3},-4)$

Inf. pts : $(-1, f(-1)) = (-1,0)$

$(1, f(1)) = (1,0)$







9. $f(x) = \dfrac{1}{x^2-16} = \dfrac{1}{(x+4)(x-4)}$

(a) D: $x \neq \pm 4$

(b) x-int: $0 = \dfrac{1}{(x+4)(x-4)}$

$0 \neq 1$

$\boxed{\text{no x-int}}$

y-int: $f(0) = \dfrac{1}{0-16} = -\dfrac{1}{16}$

(c) VA: $x=-4$, $x=4$

no holes

HA: highest powers of x $\Rightarrow y=0$

(d) $f' = \dfrac{(x^2-16)(0) - 1(2x)}{(x^2-16)^2} = \dfrac{-2x}{(x^2-16)^2}$

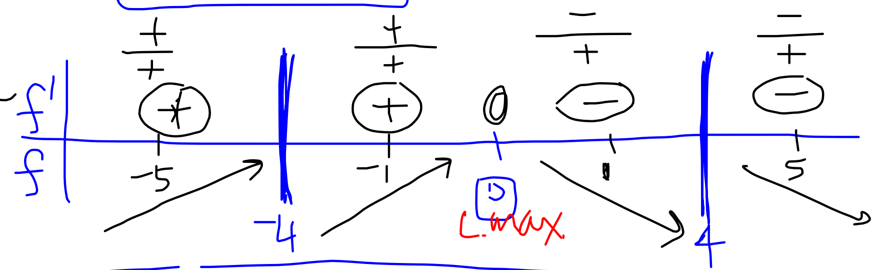
$f'=0$: $0 = \dfrac{-2x}{(x^2-16)^2} \rightarrow 0=-2x$

$x=0$

f' DNE when denom $=0 \rightarrow x=\pm 4$ (not in D. of f)

$\therefore$ $\boxed{CV: x=0}$

(e)



$\boxed{\begin{array}{l} \text{Inc}: (-\infty,-4)\ (-4,0) \\ \text{Dec}: (0,4)\ (4,\infty) \end{array}}$

(f) next page $\Rightarrow$

(g) $f'' = \dfrac{(x^2-16)^2(-2) - (-2x)\left[2(x^2-16)(2x)\right]}{(x^2-16)^4}$

$= \dfrac{-2(x^2-16)\left[(x^2-16) - 4x^2\right]}{(x^2-16)^4}$

$= \dfrac{-2\cancel{(x^2-16)}(-3x^2-16)}{(x^2-16)^{\cancel{4}\,3}} = \dfrac{(-2)(-3x^2-16)}{(x^2-16)^3}$

$f''=0 = \dfrac{-2(-3x^2-16)}{(x^2-16)^3} \rightarrow (-2)(-3x^2-16)=0$

$-3x^2-16=0$

$-3x^2=16$

$x^2 \neq -16/3$ never

f'' DNE at $x=\pm 4$ (not in Domain)

$\begin{array}{l} \text{C}\uparrow: (-\infty,-4)\ (4,\infty) \\ \text{C}\downarrow: (-4,4) \end{array}$

(h) NO Infl Pts

IMPT PTS

No x-int

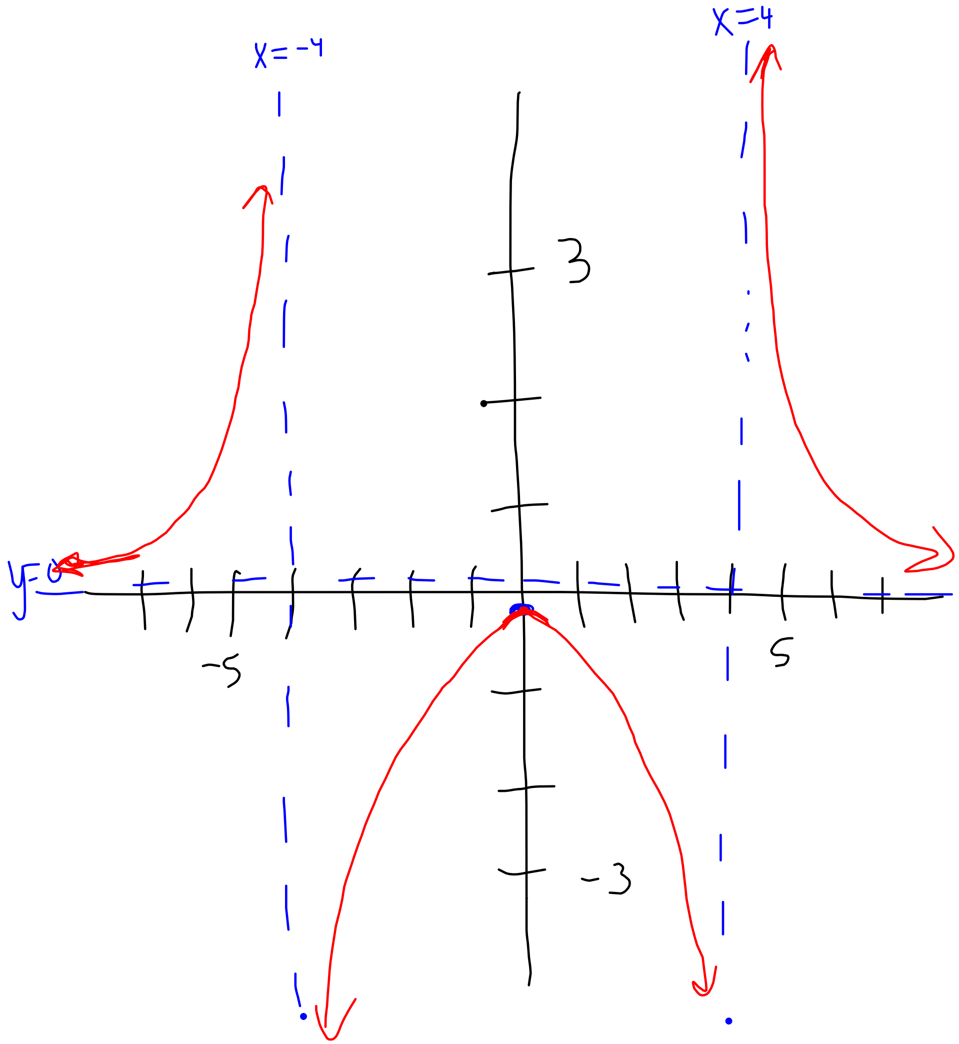
y-int: $(0, -1/16)$

VA: $x=-4$, $x=4$

no holes

HA: highest powers of x
$\Rightarrow y=0$

local max: $(0, f(0)) = (0, -1/16)$



10. $f(x) = \sqrt[3]{2x+5} = (2x+5)^{1/3}$

(a) Domain: $\mathbb{R}$

(b) x-int: $(0)^3 = \left(\sqrt[3]{2x+5}\right)^3$

$0 = 2x+5$

$-5 = 2x$

$-5/2 = x$

y-int: $f(0) = \sqrt[3]{2(0)+5} = \sqrt[3]{5} \approx 1.71$

(c) no VA/Holes

no HA

(d) $f' = \frac{1}{3}(2x+5)^{-2/3}(2)$

$= \frac{2}{3}\left(\frac{1}{(2x+5)^{2/3}}\right)$

$= \frac{2}{3}\left(\frac{1}{\left(\sqrt[3]{2x+5}\right)^2}\right) = \frac{2}{3\left(\sqrt[3]{2x+5}\right)^2}$

$f' = 0$: $\Rightarrow 0 \neq 2$

f' DNE: $2x+5 = 0$

$2x = -5$

$x = -5/2$

$\boxed{CV: x = -5/2}$

(e)

		-3		$-5/2$		0	
f'	$\frac{+}{(+)(+)}$ $(+)$			ND		$\frac{+}{(+)(+)}$ $(+)$	
f				$-5/2$ vert. tan.			

$\boxed{\begin{array}{l}\text{Inc: } (-\infty, -5/2)\ (-5/2, \infty) \\ \text{Dec. never}\end{array}}$

(f) no local ext.

(g) $f' = \frac{2}{3}(2x+5)^{-2/3}$

$f'' = -\frac{4}{9}(2x+5)^{-5/3}(2)$

$= \frac{-8}{9}\left(\frac{1}{(2x+5)^{5/3}}\right) = \frac{-8}{9\left(\sqrt[3]{2x+5}\right)^5}$

$f'' = 0$: $0 \neq -8$

f'' DNE @ $x = -5/2$

		-3		$-5/2$		0	
f''	$\frac{(-)}{(+)(-)}$ $(+)$			ND		$\frac{(-)}{(+)(+)}$ $(-)$	
f				$-5/2$ I			

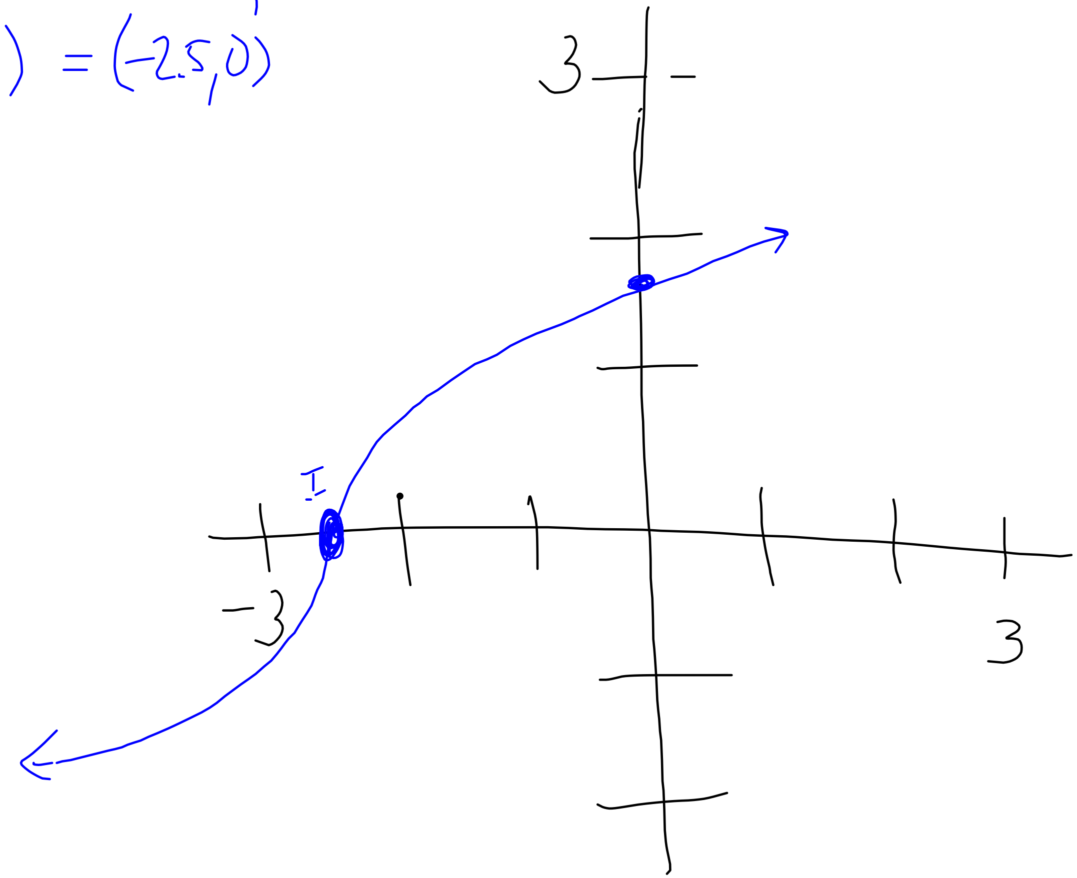
$\boxed{CA: (-\infty, -5/2)\quad CV: (-5/2, \infty)}$

• Impt pts

x-int: $(-2.5, 0)$

y-int: $\approx (0, 1.71)$

(h) Inf pt: $(-2.5, f(-2.5)) = (-2.5, 0)$



11. Sketch the graph of a function that satisfies the following conditions: Scale = 1's

Vertical Asymptotes at $x = -3, 0, 2$

Horizontal Asymptote at $y = 0$

$f'(-2) = 0$ CN $\qquad f(-2) = 1$ → pt: (−2, 1)

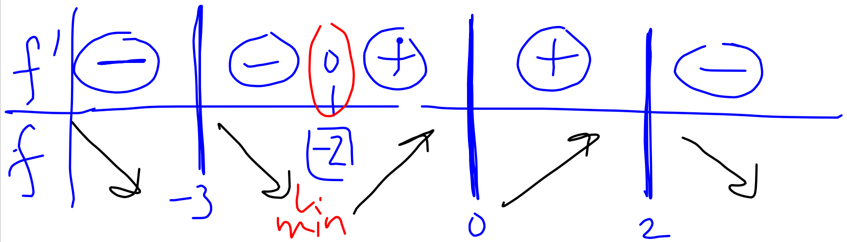
Inflection point: (1,0)

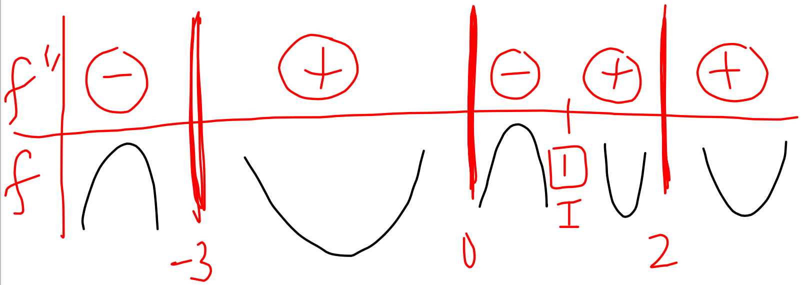
$f'(x) < 0$ on $(-\infty, -3), (-3, -2)$ and $(2, \infty)$

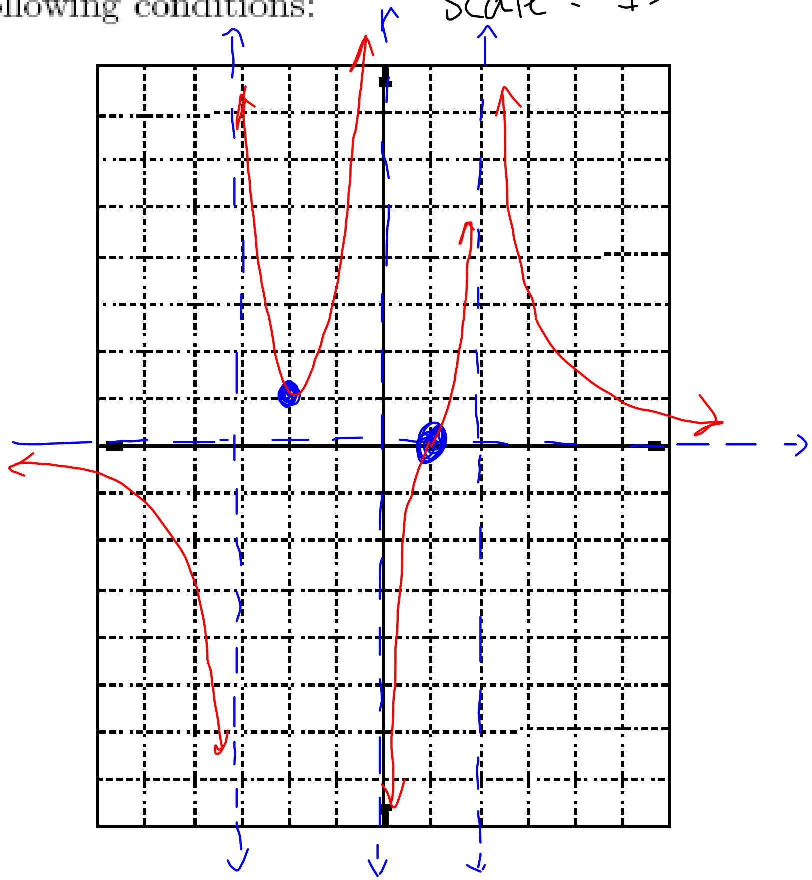
$f'(x) > 0$ on $(-2, 0)$ and $(0, 2)$

$f''(x) < 0$ on $(-\infty, -3)$ and $(0, 1)$

$f''(x) > 0$ on $(-3, 0), (1, 2)$ and $(2, \infty)$

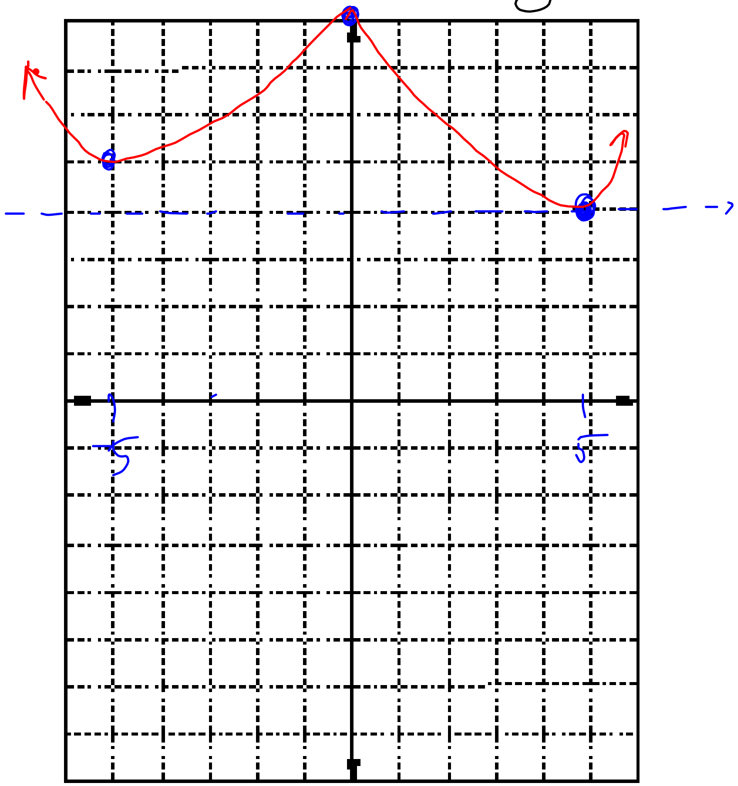






12. Sketch the graph of a function that satisfies the following conditions:

Count by 1's



Continuous for all reals (no breaks)

Domain: All reals

y-values

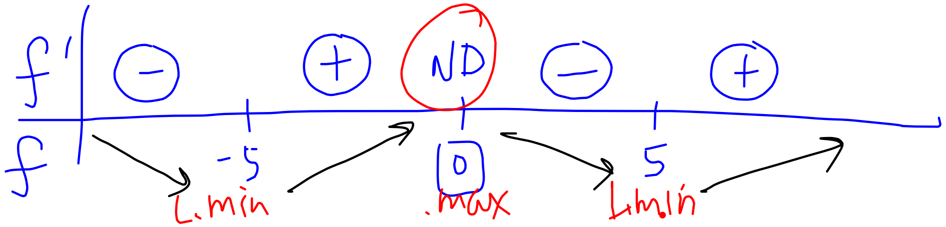
Range: All reals greater than or equal to 4

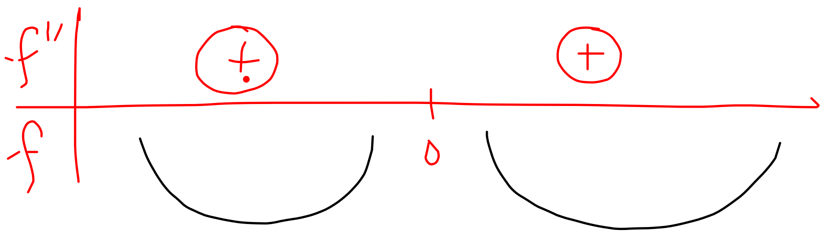
$f'(x) > 0$ on $(-5, 0)$ and $(5, \infty)$

$f'(x) < 0$ on $(-\infty, -5)$ and $(0, 5)$

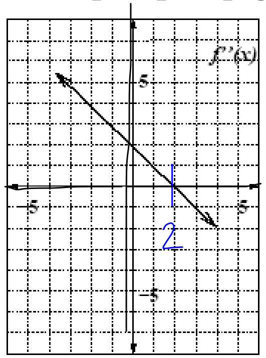
$f'(0)$ undefined

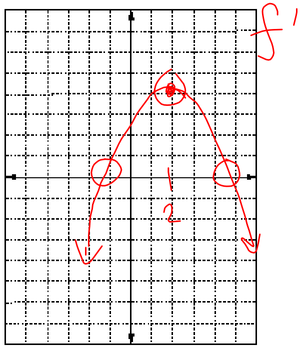
$f''(x) > 0$ on $(-\infty, 0)$ and $(0, \infty)$

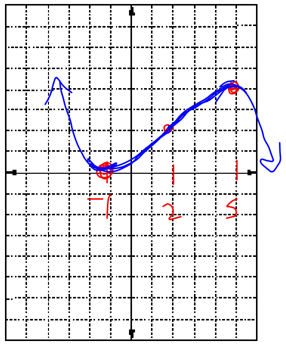




13. Using the given graph of $f''(x)$, sketch possible graphs of $f'(x)$ and $f(x)$.







f'' above x-axis (+) 0 below x-axis (−)

f' ↗ 2 ↘ L.max

f'' (+) 0 (−)

f ∪ 2 ∩

f' below x-axis (−) 0 (+) 0 (−)

f ↘ −1 ↗ 5 ↘

L. min L.max

* Lots of different graphs.