

Week-In-Review #15

1. Find the equations of the volume and surface area of a cylinder as functions of its radius and its height.

2. Determine the domain of the following functions:

(a) $f(x, y) = e^{3x} - y \ln(x + 5)$

(b) $g(x, y) = \frac{6}{2x + 3y}$

3. A company sells cameras and film to be used in the camera. Let x be the number of cameras sold per day and y be the number of rolls of film sold per day. Suppose the price of each roll of film is given by $q = 802 - 3x - 0.5y$ and the price of each camera is given by $p = 1400 - 12x - y$. Also, suppose the daily cost function for the company is given to be $C(x, y) = 15000 + 50x + 0.5y$.

- (a) Find the revenue function for the company.
- (b) Find the profit function for the company.
- (c) What is the profit/loss for the company when 5 cameras and 50 rolls of film are sold in one day?

4. Find $f_x(x, y)$ and $f_y(x, y)$ for the following.

(a) $f(x, y) = 2x^2 - 3y^2 + 5x^2y^2 - e$

(b) $f(x, y) = (7y - 3x^2)^4$

(c) $f(x, y) = \frac{x^2}{y^4 + 5}$

(d) $f(x, y) = 2e^{xy} + e^{x-y}$

(e) $f(x, y) = 4 \ln(x^2 + 3x + y^7) - \pi$

5. A company has a monthly production function given by

$$f(x, y) = 100x^{0.6}y^{0.4}$$

where x is the amount of labor used and y is the amount of capital used.

- (a) If the company is now using 15 units of labor and 23 units of capital, find the marginal productivity of labor and the marginal productivity of capital.

- (b) For the greatest increase in the company's productivity, should labor or capital be increased from its current state?

6. Find all second-order partial derivatives: f_{xx} , f_{yy} , f_{xy} , and f_{yx} .

(a) $f(x, y) = 5xy^3 - 7x^3y$

(b) $f(x, y) = \frac{x^2}{y}$

(c) $f(x, y) = xe^{xy}$

7. Find all of the critical points for $f(x, y) = 3x^2 + 5y^2 - 8xy + 2x + 6y + 3$

8. Locate any critical points of the following and, if possible, identify each as local extrema or a saddle point.

(a) $f(x, y) = 2x^2 + 5y^2 + 6x - 2y + 12$

$$(b) f(x, y) = -3x^2 + xy - y^2 - 4x - 3y$$

$$(c) f(x, y) = xy - x^3 - y^2$$

9. A firm manufactures and sells two products, X and Y , that sell for \$15 and \$10 each, respectively. The cost of producing x units of X and y units of Y is

$$C(x, y) = 400 + 7x + 4y + 0.01(3x^2 + xy + 3y^2)$$

Find the values of x and y that maximize the firm's profit.

10. Continental Airlines has the restriction that a carry-on bag have a maximum combined linear measurement ($L+W+H$) of 51 inches. What are the dimensions of the largest bag, in volume, that will meet this requirement?