Week-In-Review #2 (1.5 and 3.1)

1. Are the following one-to-one functions?

2. Find the domain of the following functions. Write your answer using interval notation.

   (a) \( f(x) = \log_8(5x - 9) \)

   (b) \( g(x) = \frac{3x^2}{\log(x + 2)} \)

3. Rewrite the following in logarithmic form:

   (a) \( 2^5 = 32 \)

   (b) \( e^0 = 1 \)

4. Write the following as a single logarithm: \( \frac{1}{3}(\log_b x - \log_b y + 2\log_b z) \)
5. Write the following as a sum, difference, and/or multiple of logarithms: \( \ln \left( \frac{3x(x + 1)}{(2x + 1)^2} \right) \)

6. If \( \log_b 3 = 1.578 \) and \( \log_b 7 = 2.638 \), evaluate \( \log_b \left( \frac{49b}{3} \right) \).

7. Solve the following for \( x \) algebraically. Give your solution as an EXACT answer.
   (a) \( 2e^{2x} + 5e^x = 3 \)
   (b) \( 3e^{4x} - 1 = 7 \)
   (c) \( 2 \cdot 7^{2x} = 215 \)
(d) \( \ln (\log_3 (2x + 7)) = 0 \)

(e) \( \log (2x + 4) + \log (x + 6) = 1 \)

(f) \( \log_4 (x + 8) = 2 + \log_4 (x - 7) \)

(g) \( \log_b x = \frac{1}{2} \log_b 9 - \log_b 4 \) where \( b > 0 \) and \( b \neq 1 \)
8. How long will it take for your money to triple in an account paying 8% annual interest compounded continuously?

9. Use the given graph of $f(x)$ to answer the following questions.

(a) $\lim_{x \to -1^-} f(x) = \quad$ (b) $\lim_{x \to -1^+} f(x) = \quad$ (c) $\lim_{x \to -1} f(x) = \quad$

(d) $\lim_{x \to -3^+} f(x) = \quad$ (e) $\lim_{x \to 0} f(x) = \quad$ (f) $\lim_{x \to 1^+} f(x) = \quad$

(g) $\lim_{x \to -5} f(x) = \quad$ (h) $f(-1) = \quad$; $f(-5) = \quad$

(i) List any points of discontinuity and state which condition of continuity is violated at each of these points.

10. Numerically estimate the following limit: $\lim_{x \to 2} \frac{x - 2}{(x - 2)^2}$, if it exists.

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11. Given \( f(x) = \begin{cases} 
  x + 3 & , \ x < -1 \\
  0 & , \ x = -1 \\
  x^2 + 1 & , \ x > -1 
\end{cases} \), algebraically find the following. If a limit does not exist, explain why not.

(a) \( \lim_{x \to -1^-} f(x) \)

(b) \( \lim_{x \to -1^+} f(x) \)

(c) \( \lim_{x \to -1} f(x) \)

(d) \( \lim_{x \to -3} f(x) \)

(e) \( \lim_{x \to 5} f(x) \)

12. Algebraically find each of the following, if it exists. If a limit does not exist, explain why not.

(a) \( \lim_{x \to 1} \frac{2x^2 + 5x - 4}{x + 2} \)

(b) \( \lim_{h \to 0} \frac{2x + 3h}{x + 1} \)

(c) \( \lim_{x \to 0} \frac{x^4}{x^3 + 2} \)

(d) \( \lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5} \)
(e) \[ \lim_{x \to -1} \frac{x + 2}{x^2 + 3x + 2} \]

(f) \[ \lim_{x \to 3} \frac{x - 3}{x^2 - 9} \]

(g) \[ \lim_{x \to -1} \frac{\sqrt{x + 5} - 2}{x + 1} \]

(h) \[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \] if \( f(x) = \frac{1}{x - 1} \)
(i) \( \lim_{x \to 0} \frac{x + 2e^x}{e^x + 1} \)

(j) \( \lim_{x \to 2} \frac{2x^2 - 8}{|x - 2|} \)

(k) If \( f(x) = \begin{cases} 
2x^2 + 1, & x < 0 \\
x^2 + 1, & x \geq 0 
\end{cases} \), find \( \lim_{x \to 0} f(x) \)
(l) Given \( f(x) = \begin{cases} 2x^2 - 8, & x < 1 \\ -4\sqrt{x} - 2, & x > 1 \end{cases} \), find

(i) \( \lim_{x \to 1} f(x) \)

(ii) \( \lim_{x \to 4} f(x) \)

(iii) \( f(1) \)

13. Determine the interval(s) where the following functions are continuous.

(a) \( f(x) = \frac{x + 7}{(x + 7)(x - 9)} \)

(b) \( g(x) = x^{100} + x^{-1} \)

(c) \( h(x) = 2e^x + 5 \)
14. Where is \( f(x) = \begin{cases} 
  x - 6 & \text{, } x \leq 4 \\
  \frac{x + 1}{(x - 5)(x + 1)} & \text{, } x > 4
\end{cases} \) discontinuous?

15. Find the value(s) of \( a \) and \( b \), if any exist, that will make \( g(x) = \begin{cases} 
  5a - x & \text{, } x < 2 \\
  6 & \text{, } x = 2 \\
  bx^2 + 4 & \text{, } x > 2
\end{cases} \) continuous on \((-\infty, \infty)\).