Week-In-Review #4 (4.1, 4.2, 4.3)

1. Differentiate the following.

   (a) \( f(x) = x^2 + 7x^3 - \pi x + 6 - \ln x \)

   (b) \( g(t) = 2e^t + \sqrt[3]{t^3} - \frac{1}{t^3} + et \)

   (c) \( h(x) = \frac{4x^3 - 7x + 2}{x^{4/7}} \)

   (d) \( r(y) = 5y^2 \left( 2\sqrt{y} + \frac{3}{y^5} - 7 \right) \)

   (e) \( k(x) = (7x^4 - 10x + \sqrt{x} - e^2) \left( \frac{1}{x} - \frac{2}{\sqrt{x}} \right) + 8 \)

   (f) \( y = \frac{(5t^4 + \sqrt[8]{t^8} - 2t)}{(3t^6 - 9 + 5\ln t)} \)
(g) \( m(n) = 3(8 - 4n^2)^3 \)

(h) \( n(x) = \frac{10}{\sqrt{7} + x} \)

(i) \( g(x) = -2\sqrt[3]{(x^2 + 4)^2 + xe^x} \)

(j) \( y = (3h - 4h^7)^2(2\sqrt{h} - 6)^3 \)
(k) \[ f(x) = \frac{(2x^8 - 6x^5 + 1)^4}{(9x - 7)(-5x + 3)^2} \]

(l) \[ y = \sqrt[3]{(2x + 5)^4(3x^2 - 7)^6} \]

2. Find the value(s) of \( x \) where \( f(x) = \frac{x}{x^2 + 4} \) has a horizontal tangent line.
3. Find the equation of the line tangent to the following functions at the indicated point.

(a) \( f(x) = 5\sqrt{6x^2} + 4 \) at \( x = 0 \)

(b) \( g(x) = x^2(x - 1)^3 \) at \((2, 4)\)

4. If \( m(r) = (r^2 + 3)^4 \) and \( r(t) = e^t - \frac{1}{t^5} \), find \( \frac{dm}{dt} \).
5. Find \( h'(0) \) for the functions below, where \( f(x) \) and \( g(x) \) are unspecified differentiable functions with the following given information:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
<th>( f'(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>9</td>
<td>-2</td>
<td>-1</td>
</tr>
</tbody>
</table>

(a) \( h(x) = 2f(x) - 3x^2g(x) \)

(b) \( h(x) = f(g(x)) \)

(c) \( h(x) = g(f(x)) \)

(d) \( h(x) = e^x f(x)g(x) \)
6. Use the graph given below to find the following values.

(a) Find \( h'(-4) \), if \( h(x) = f(x)g(x) \).

(b) Find \( h'(2) \), if \( h(x) = \frac{f(x)}{g(x)} \).
7. A ball is thrown vertically upward from the ground at a velocity of 64 ft/sec. Its distance from the ground at \( t \) seconds is given by \( s(t) = -16t^2 + 64t \).

(a) Find the velocity function, \( v(t) \).

(b) How fast is the ball moving 2 seconds after being thrown? 3 seconds after being thrown?

(c) How long after the ball is thrown does it reach its maximum height?

(d) How high will the ball go?

8. The total cost (in dollars) of producing \( x \) items is given by \( C(x) = 500 + 40x + 0.05x^2 \).

(a) Find the marginal cost function.

(b) Use the marginal cost function to estimate the cost of producing the 100\(^{th} \) item.

(c) Find the exact cost of producing the 100\(^{th} \) item.

(d) Use the marginal cost function to estimate the total cost when 100 items are produced.
9. A particular book is sold at a price according to the price-demand function, \( p(x) = -0.5x + 50 \). It costs a total of $650 to produce 75 of these books and there are $275 of fixed costs associated with producing the books. If \( x \) represents the number of these books that are made and sold, find the following.

(a) \( C(x), R(x), \) and \( P(x) \)

(b) Marginal cost, marginal revenue, and marginal profit functions

(c) \( R'(10) \) and interpret.