Week-In-Review #5 (4.4, 4.5, 5.1)

1. Find the derivative of each of the following functions. DO NOT SIMPLIFY.

   (a) \( g(x) = 2e^{\sqrt{x}+10x^2-6} \)

   (b) \( h(x) = e^{\frac{3}{\sqrt{x}-14x^2}} \)

   (c) \( k(x) = 7xe^{x^8+1} \)

   (d) \( y = 3x^2e^{(x^2+3)^4(x+6)^{10}} \)

   (e) \( m(x) = \frac{(e^x + e^{-x})^5}{3x^4 + 8} \)

   (f) \( g(x) = 2^x + x^2 \)
\( n(x) = 3^{-x + 6\sqrt[3]{x}} + e^{2x} + 7 \)

\( f(x) = \ln (x^2 + 4x^6)^3 \)

\( y = \ln \left( \frac{(3x^4 + 9)^6}{\sqrt{x} + 17} \right) \)

\( f(x) = \log_5 (3x^5 - 4x^2 + 3) + \log_6 2 \)

\( g(x) = 4 \log_2 (x^3(1 + 9x^4)) \)
(l) $h(x) = (\log_7 (5x))^3$

(m) $y = \log_3 (\ln (\log_4 (x^2)))$

2. Find the equation of the line tangent to $f(x) = \log x$ at $x = 1$.

3. Given $f(x) = 25x + e^{-x}$, find
   (a) the relative rate of change of $f(x)$.

   (b) the percentage rate of change of $f(x)$. 
4. Given the price-demand function, \( p = \sqrt{1200 - x^2} \), find the demand as a function of price.

5. If the price-demand equation for a product is given to be \( x = 5(p - 60)^2 \), determine whether the demand is elastic, inelastic, or has unit elasticity at the indicated prices.

(a) \( p = $10 \)

(b) \( p = $20 \)

(c) \( p = $25 \)
6. Given demand for a product is \( x = (225 - 5p)^{1/2} \),

(a) Classify the type of elasticity at the current price of $10.

(b) If the price decreases by 10% from the current price, what is the approximate change in demand?

(c) Should the price be raised or lowered from the current price of $10 in order to increase revenue?

(d) If the price in another area is $40 and increases by $2, what is the approximate change in demand?

(e) What price maximizes revenue?
7. For each of the following functions, use calculus to find any critical values, the intervals where the function is increasing/decreasing, and any points of relative extrema (classifying the type).

(a) \( f(x) = \frac{-1}{x^2 - 16} \)

(b) \( f(x) = \frac{\ln x}{x} \)
(c) \( g(x) = e^x - e^{-x} \)

8. Given \( f(x) \) is a continuous function with \( f'(x) = (n)(x + 2)(x - 3)^4(x + 5)^3 \), where \( n \) is a function that is always negative, determine critical values of \( f(x) \), intervals where \( f(x) \) is increasing or decreasing and any values of \( x \) where relative extrema of \( f(x) \) will occur.

9. The price-demand for a product is given by \( p = -0.2x + 16 \) and the cost of making the product is given by \( C(x) = 5 + 8x \). Use calculus to determine when the profit from the sale of this product is at a maximum.
10. Use the graph below to answer the questions that follow.

(a) If the given graph is that of \( f(x) \), where is \( f'(x) > 0? \) \( f'(x) < 0? \) \( f'(x) = 0? \)

(b) If the given graph is that of \( f(x) \), at which \( x \)-values do relative extrema occur?

(c) If the given graph is that of \( f'(x) \) and \( f(x) \) is a continuous function on \((-\infty, \infty)\), what are the critical values of \( f(x) \)?

(d) If the given graph is that of \( f'(x) \) and \( f(x) \) is a continuous function on \((-\infty, \infty)\), where is \( f(x) \) decreasing? \( f(x) \) increasing?

(e) If the given graph is that of \( f'(x) \) and \( f(x) \) is a continuous function on \((-\infty, \infty)\), at which \( x \)-values do relative extrema occur in the graph of \( f(x) \)?