Exam III Review (Sections 6.1-6.6, 6.7 topic, and 8.1-8.2)

Note: This collection of questions is intended to be a brief overview of the exam material (with an emphasis at the beginning on material from Sections 6.7, 8.1 and 8.2, which I have not previously reviewed). This is not intended to represent an actual exam. When studying you should also rework your notes, the previous week-in-reviews for Exam III material, and be familiar with your suggested and online homework problems.

1. Determine the consumers’ surplus and producers’ surplus when the marketplace is acting at equilibrium for demand and supply given to be the following:

(a) \( D(x) = 4500 - 0.04x^2, S(x) = 0.03x^2 + 1700 \)

(b) \( D(x) = 700e^{-0.05x}, S(x) = 50e^{0.06x} \)
2. Determine the domain of the following functions:

(a) \( f(x, y) = e^{3x} - y \ln (x + 5) \)

(b) \( g(x, y) = \frac{6}{2x + 3y} \)

3. A company sells cameras and film to be used in the camera. Let \( x \) be the number of cameras sold per day and \( y \) be the number of rolls of film sold per day. Suppose the price of each roll of film is given by \( q = 802 - 3x - 0.5y \) and the price of each camera is given by \( p = 1400 - 12x - y \). Also, suppose the daily cost function for the company is given to be \( C(x, y) = 15000 + 50x + 0.5y \).

(a) Find the revenue function for the company.

(b) Find the profit function for the company.
4. Find $f_x(x, y)$ and $f_y(x, y)$ for the following.

(a) $f(x, y) = 2x^2 - 3y^2 + 5x^2y^2 - e$

(b) $f(x, y) = (7y - 3x^2)^4$

(c) $f(x, y) = \frac{x^2}{y^4 + 5}$

(d) $f(x, y) = 2e^{xy} + e^{x-y}$

(e) $f(x, y) = 4\ln(x^2 + 3x + y^7) - \pi$
5. Find all second-order partial derivatives: $f_{xx}, f_{yy}, f_{xy},$ and $f_{yx}$.

(a) $f(x, y) = 5xy^3 - 7x^3y$

(b) $f(x, y) = \frac{x^2}{y}$

(c) $f(x, y) = xe^{xy}$
6. Find the exact value of \( \int_1^2 \left( \frac{6 \ln x}{x} \right) \, dx \)

7. Find the value(s) of \( a \) if \( \int_2^a 2x^3 \, dx = \frac{65}{2} \) and \( a > 0 \).

8. Determine the average value of \( f(x) = 2x^5 - 6x + 7 \) on the interval \([-3, 4]\).
9. Kathryn deposits $1500 into a bank account that earns 3.0% annual interest compounded continuously. Determine Kathryn’s average balance over the first 5 years.

10. Kevin deposits $500 into an account where the rate of change of the amount in the account is given by \(23.75e^{0.0475t}\) dollars/year, \(t\) years after the initial deposit. Determine by how much the account changed during the third and fourth years.

11. Given the graph of \(g(x)\),

   (a) will \(\int_3^6 g(x) \, dx\) be positive or negative? (Circle one.)

   (b) if you are told that \(\int_{-3}^6 2g(x) \, dx = -12\) and \(\int_{-3}^{0} g(x) \, dx = 2\), find the AREA of C.

   (c) set-up the integral(s) representing the total area of A and B combined?
12. Find the area between the following functions on the given intervals.

(a) \( f(x) = 2x + 5, \ g(x) = -3x - 10 \) on \([-5, 5]\)

(b) \( f(x) = \frac{2}{x}, \ g(x) = \ln x \) on \([3, 6]\)
13. Find the area bounded by $f(x) = x^3 - 4x$ and the $x$-axis.

14. KNB Inc. determines the marginal revenue to produce $x$ widgets is given by $R'(x) = 50 - 0.5x$, while the marginal cost is given by $C'(x) = \frac{100}{x + 1} + 0.6x$, where both marginals are in dollars per widget. Determine the total change in profit from $x = 10$ to $x = 40$. 
15. Find the cost function for a tape manufacturer, if the marginal cost, in dollars/case, is given by $150 - 0.1e^x$, where $x$ is the number of cases of tape produced and the manufacturer has $\$100$ worth of fixed costs.

16. If $f'''(x) = 6x + 10$, $f'(1) = -1$, and $f(1) = -8$, find the $x$-intercepts of $f(x)$. 
17. Given $\int_{2}^{5} (-x^2 + 4) \, dx$

(a) Sketch the region indicated by this integral.

(b) Use the order properties of integrals to find bounds for the value of the given integral.

(c) Approximate the value of the integral by finding the left hand and right hand Riemann sums with 4 rectangles. Draw pictures of the areas being found with 4 rectangles.

(d) Find the exact value of the integral.

(e) How much total area is there between the curve $y = -x^2 + 4$ and the $x$-axis from $x = 2$ to $x = 5$?
18. Evaluate the following:

(a) \[ \int \left( 2x + \frac{5}{x^3} - x \sqrt{x} - e \right) \, dx \]

(b) \[ \int \frac{y^2 - \sqrt{y}}{y^3} \, dy \]

(c) \[ \int e^{4t} \, dt \]

(d) \[ \int \frac{3}{5x \cdot \ln 4x} \, dx \]
(e) \[ \int (10x - 20)e^{x^2-4x} \, dx \]

(f) \[ \int t\sqrt{2t + 5} \, dt \]

(g) \[ \int \frac{y + 2.5}{(y^2 + 5y + 6)^3} \, dy \]