

**Exam II: Night Before Drill**

*Note:* This collection of questions is intended to give you an idea of different types of questions that might be asked on the exam. This is not intended to represent an actual exam. These questions cover Chapters 3 and 4 in *Calculus for Business, Economics, Life Sciences, and Social Sciences* by Barnett, Ziegler, and Byleen.

**Find the following limits.**

1. 
$$\lim_{x \rightarrow 2} \frac{2x^2 - 4}{3x^2 - 1}$$

2. 
$$\lim_{x \rightarrow 0} \frac{4x^3 - x^2}{2x^2 - x}$$

3. 
$$\lim_{x \rightarrow 5} \frac{\sqrt{x} - 5}{x - 25}$$

4. 
$$\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$$

5. 
$$\lim_{x \rightarrow \infty} \frac{3x^4 - x + x^3}{4x^3 - x^2 + 5x^4}$$

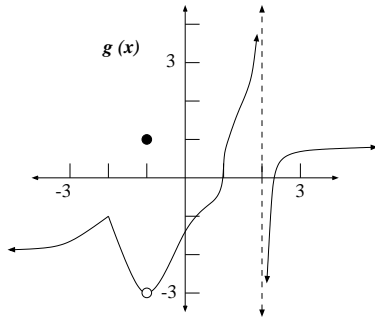
6. 
$$\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{x - 6}$$

7. 
$$\lim_{x \rightarrow -\infty} \frac{x + x^2}{2x^4 - x^2}$$

8. 
$$\lim_{x \rightarrow \infty} \frac{e^x - 1}{e^{-x} + 2e^x}$$

9. 
$$\lim_{x \rightarrow 1} f(x) \text{ if } f(x) = \begin{cases} 2x^2 - 8 & , x < 1 \\ -4\sqrt{x} - 2 & , x > 1 \end{cases}$$

Use the graph below to answer #10 - #18 .



10. Find  $\lim_{x \rightarrow -1} g(x)$

11. Find  $\lim_{x \rightarrow 2} g(x)$

12. Find  $\lim_{x \rightarrow 1} g(x)$

13. Find  $\lim_{x \rightarrow 2^+} g(x)$

14. Find  $\lim_{x \rightarrow 2^-} g(x)$

15.  $\lim_{x \rightarrow -\infty} g(x)$

16.  $\lim_{x \rightarrow \infty} g(x)$

17. Find all values of  $x$  where  $g(x)$  is discontinuous and state the continuity condition that is violated.

18. Find all values of  $x$  where  $g(x)$  is not differentiable and give a reason why it is not differentiable.

19. Where is  $f(x) = \begin{cases} x - 6 & , x \leq 4 \\ \frac{x+1}{x-3} & , x > 4 \end{cases}$  discontinuous?

20. Given  $f(x) = x^{1/3}$ , where is  $f(x)$

(a) continuous?

(b) not differentiable?

21. Find the average rate of change of the function  $f(x) = \frac{1}{x}$  from  $x = 2$  to  $x = 10$ .

22. Use the limit definition of derivative to find the derivative of the following functions:

(a)  $f(x) = 2x^2 - x$

(b)  $g(x) = \frac{x}{x+1}$

(c)  $k(x) = \sqrt{x-3}$

Find the derivatives of the following functions:

23.  $f(x) = x^3 - \frac{1}{x^2} + 2\sqrt{x} - e^2 + 4$

24.  $f(x) = (x^2 + 6x + 1)^4$

25.  $f(x) = (2x + 1)\sqrt{x^2 + 1}$

26.  $f(x) = \frac{3}{\sqrt[5]{0.5x^7 + 9}}$

27.  $f(x) = \frac{x + 1}{(x - 2)^3}$

28.  $f(x) = \sqrt[3]{3x^2(2x - 8x^6)^4}$

29. Find the instantaneous rate of change of  $f(x) = \sqrt{(x+7)^3}$  at  $x = 0$ .
30. Find the equation of the tangent line to  $f(x) = x^3 - 5x^2 + \sqrt{x}$  at  $x = 1$ .
31. If  $C(x) = 5x^2 - 2x + 1000$  is the cost of producing  $x$  items, in dollars, and  $p = -3x + 30$  is the price-demand function for the items, find the following:
- (a)  $R(x)$
  - (b)  $P(x)$
  - (c) Marginal profit
  - (d) The approximate profit from selling the 11<sup>th</sup> item
  - (e) Average profit
  - (f) Marginal average profit
32. If  $R(x)$  represents a company's revenue function, what does  $R'(5)$  represent?

33. Given  $f(x) = \frac{2x + 5}{x^2}$ ,  $f'(x) = \frac{-2(x + 5)}{x^3}$ , and  $f''(x) = \frac{2(2x + 15)}{x^4}$ , use calculus to find the following and then sketch  $f(x)$ :

(a) The critical value(s) of  $f(x)$

(b) Intervals where  $f(x)$  is increasing/decreasing

(c) The point(s) where any local extrema of  $f(x)$  occur - indicate whether your point is a max or min.

(d) Intervals where  $f(x)$  is concave up/concave down

(e) The  $x$ -values where any inflection points of  $f(x)$  occur

(f) The  $x$  and  $y$ -intercepts of  $f(x)$

(g) Any asymptotes of  $f(x)$  (vertical and horizontal)

34. Determine the absolute extrema, if any exist, of each function on the indicated intervals.

(a)  $f(x) = x^3 - 3x^2 - 24x + 5$

i. On  $[0, 6]$

ii. On  $(-\infty, \infty)$

(b)  $f(x) = 12 - x - \frac{9}{x}$

i. On  $[-5, 5]$

ii. On  $(-10, 0)$

35. From a 27-inch by 9-inch piece of cardboard, square corners are cut out so that the sides can be folded up to form a box with no top. What is the maximum volume the box will be able to hold?

36. A farmer has 1200  $m$  of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river.
- (a) Find the fence dimensions that will maximize the enclosed area.
  - (b) What is the maximum enclosed area?

37. A bus company charged \$60 per person for a sight-seeing trip and obtained 40 people for the trip. The company has data that indicates that for the same trip, each \$2 increase in the price above \$60 results in a loss of one customer. The bus company has fixed costs of \$3000 for each trip and additional costs of \$4 per customer.
- (a) What should the company charge in order to maximize revenue?
  - (b) What should the company charge in order to maximize profits?