

**Exam III: Night Before Drill**

*Note:* This collection of questions is intended to give you an idea of different types of questions that might be asked on the exam. This is not intended to represent an actual exam. These questions cover Chapters 5, 6 and 7 in *Calculus for Business, Economics, Life Sciences, and Social Sciences* by Barnett, Ziegler, and Byleen.

1. How long will it take for an investment earning 5.4% interest, compounded continuously, to triple?

2. Find  $f[g(x)]$  and  $g[f(x)]$  given

(a)  $f(x) = e^{x+2}$ ;       $g(x) = \sqrt{x+7}$

(b)  $f(x) = x^2 + 8$ ;       $g(x) = \ln x$

3. Find the interval(s) where  $f(x) = e^{x^3+2x^2-4x}$  is increasing/decreasing.

4. Find intervals, if any exist, where the following functions are concave up/concave down and determine any points of inflection.

(a)  $f(x) = \ln x^2$

(b)  $f(x) = 5xe^x$

5. The cost of producing  $x$  units of a product is given by  $C(x) = 800 + 200x - 100 \ln x$  for  $x \geq 1$ . Find the minimum average cost.

6. Find  $\frac{dy}{dx}$  given the following:

(a)  $y = x^2 3^{x^2+4x} + 3^2$

(b)  $y = \log(\ln(x^2 + 9)) + \ln 5$

(c)  $y = e^u; u = -2x^2$

7. Given the price-demand equation  $x = 10(p - 9)^2$ ,

(a) Find the elasticity of demand,  $E(p)$ .

(b) If  $p = \$5$ , is demand elastic, inelastic, or is there unit elasticity?

(c) Should the price be raised, lowered, or kept at \$5 in order to increase revenue?

(d) At what price will revenue be maximized?

8. Evaluate the following:

(a)  $\int (2x + \frac{5}{x^3} - x\sqrt[4]{x} - e) dx$

(b)  $\int \frac{y^2 - \sqrt{y}}{y^3} dy$

(c)  $\int e^{4t} dt$

(d)  $\int \frac{3}{5x \cdot \ln 4x} dx$

$$(e) \int (10x - 20)e^{x^2 - 4x} dx$$

$$(f) \int t\sqrt{2t + 5} dt$$

$$(g) \int \frac{y + 2.5}{(y^2 + 5y + 6)^3} dy$$

9. Find the cost function for a tape manufacturer, if the marginal cost, in dollars/case, is given by  $150 - 0.1e^x$ , where  $x$  is the number of cases of tape produced and the manufacturer has \$100 worth of fixed costs.

10. Given  $\int_2^5 (-x^2 + 4) dx$

- (a) Sketch the region indicated by this integral.
- (b) Approximate the value of the integral by finding the left hand and right hand Riemann sums with 4 rectangles and then again with 100 rectangles. Draw pictures of the areas being found with 4 rectangles.
- (c) Find the exact value of the integral.
- (d) How much area is there between the curve  $y = -x^2 + 4$  and the  $x$ -axis from  $x = 2$  to  $x = 5$ ?

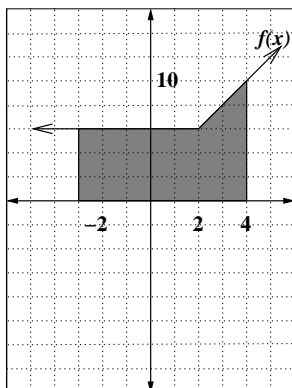
11. Evaluate the following EXACTLY:

(a)  $\int_a^b \left(x^3 + \frac{1}{x}\right) dx$

(b)  $\int_2^4 (e^t + \pi) dt$

(c)  $\int_1^3 \frac{x^2 + 1}{x^3 + 3x} dx$

12. Write a definite integral to indicate the shaded area in the graph below.



13. Suppose copper is being extracted from a mine at a rate given by  $y = 100e^{-0.2t}$ , where  $t$  is the number of years since mining began and  $y$  is measured in tons of copper/year. At this rate, how much copper will be extracted during the third year of mining?

14. If the temperature  $C(t)$  in an aquarium is made to change according to  $C(t) = t^3 - 2t + 10$  for  $0 \leq t \leq 2$  (in degrees Celsius), what is the average temperature over the period of time for which the temperature is regulated?

15. Find the area between  $y = x^2 + 1$  and  $y = -x^2 + 19$  on  $[0, 5]$ .

16. Find the area bounded by  $y = x^3$  and  $y = x$ .

17. If supply and demand for a product are given by  $p = 5 + 0.004x^2$  and  $p = 25 - 0.004x^2$ , respectively, find the following at equilibrium price:

(a) consumers' surplus

(b) producers' surplus

18. The probability that a particular doctor will spend  $t$  hours with a patient during an office visit is given by the probability density function  $f(t) = \begin{cases} \frac{4}{3}(t+1)^{-2} & , 0 \leq t \leq 3 \\ 0 & , \text{otherwise} \end{cases}$

- (a) What is the probability that this doctor will spend more than 1 hour with a randomly selected patient?
- (b) What is the probability that this doctor will spend exactly 1 hour with a randomly selected patient?

19. Starting at age 25, you deposit \$1500 a year into a retirement account. If the deposits are treated as a continuous income stream and the money in the account earns 6% compounded continuously,

- (a) How much will you have in the account if you retire at age 65?
- (b) How much of the final amount is interest?