

Exam II Review Problems

Note: Not every topic is covered in this review.

Please also take a look at the previous Week-in-Reviews for more practice problems.

1. You have completely forgotten the combination of your lock. It is one of those “standard” combination locks, which uses a three number combination with each number in the range of 0 through 39. All you remember is that the second number is either 27 or 37, while the third number ends in a 5. In desperation, you decide to go through all possible combinations. Assuming that it takes about 10 seconds to try each combination, what is the longest possible time it can take to open your locker?
2. Many U.S. license plates display a sequence of three letters followed by three digits.
 - (a) How many such license plates are possible?
 - (b) In order to avoid confusion of letters with digits, some states do not use the letters I, O or Q on their license plates. How many of these license plates are possible?
3. Bill and Sue and four of their friends go to the movies. They all sit next to each other in the same row. How many ways can this be done if
 - (a) Sue and Bill must sit next to each other?
 - (b) Sue must not sit next to Bill?
 - (c) Sue sits on one end of the row and Bill sits on the other end of the row?

4. Susie has 30 different songs (15 rock, 5 rap, 6 country, and 4 classical) she wants to arrange on her MP3 player.
- (a) How many different playlists of all her songs are possible?

 - (b) How many different playlists are possible if all songs of the same genre must be grouped together?
5. A jewelry store chain with 8 stores in Georgia, 12 in Florida, and 10 in Alabama is planning to close 10 of these stores.
- (a) In how many different ways can this be done?

 - (b) The company decided to close 2 stores in Georgia, 5 in Florida, and 3 in Alabama. In how many different ways can this be done?
6. How many 4-person committees are possible from a group of 9 people if
- (a) There are no restrictions?

 - (b) Both Jim and Mary must be on the committee?

 - (c) Either Jim or Mary (but not both) must be on the committee?

7. You have a box that contains 3 red, 4 black, 2 green, and 5 purple balls. If you take a sample of three balls from the box, in how many ways can you get

(a) Exactly 2 black balls and one green ball?

(b) No purple balls?

(c) At least two purple balls?

(d) Exactly 2 red balls or exactly one purple ball?

8. How many different 20-digit numbers can be formed from two 5's, three 2's, five 1's, six 4's, two 3's, an 8 and a 9?

9. An author is writing a new book. She has a list of 20 different female names and 15 different male names, from which she will choose her 10 character names. In how many different ways can she

(a) Choose her 10 characters?

(b) Choose half female and half male characters?

(c) Choose exactly 6 female characters and determine the order of appearance of all the characters in her book?

10. From a group of 15 chefs, one executive chef, one pastry chef and 4 sous chefs must be chosen. Assuming all 15 chefs are qualified for all the positions, in how many different ways can the 6 chefs be chosen?

13. The following system of inequalities are constraints in a linear programming problem. Graph the feasible region. Label all lines and corner points.

$$\begin{aligned}x - y &\geq 0 \\x + y &\leq 10 \\10x + 9y &\geq 45 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

- (a) At what point is the objective function $f = 5x + 4.5y$ maximized on this region and what is the maximum value? (If not possible, explain why not.)

- (b) At what point is the objective function $f = 5x + 4.5y$ minimized on this region and what is the minimum value? (If not possible, explain why not.)

14. **SET UP** the following **Linear Programming** problem. **DO NOT SOLVE.**

You manage an ice cream factory that makes three flavors: Vanilla, Chocolate, and Strawberry. Into each batch of Vanilla go two eggs, one cup of milk and two cups of cream. Into each batch of Chocolate go one egg, one cup of milk and two cups of cream, while into each batch of Strawberry go one egg, two cups of milk and two cups of cream. You have in stock 220 eggs, 120 cups of milk, and 200 cups of cream. You make a profit of \$3 on each batch of Vanilla, \$2 on each batch of Chocolate, and \$4 on each batch of Strawberry. How many batches of each flavor should you make in order to maximize your profit?

15. A company is selling two perfumes, A and B , for \$20 and \$17 per ounce, respectively. It takes the company 3 hours and \$12 to produce each ounce of perfume A and 1 hour and \$15 for each ounce of perfume B . If the company has a total of 90 hours and \$600 for production, and the company is not allowed to make more than 20 ounces of perfume A , how many ounces of each perfume should the company produce in order to maximize its revenue? What is its maximum revenue? Is anything leftover at the optimal production level?

16. True or False? $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 1, 2, 3, 4, 5\}$.

T	F	$\emptyset \in A$		T	F	$n(A) = 5$		T	F	$n(\{3, 4\}) = 2$
T	F	$\emptyset \subseteq A$		T	F	$\{1, 3, 5\} \in A$		T	F	$n(\emptyset) = 1$
T	F	$\{1, 2, 3\} \subset A$		T	F	$2 \in A$		T	F	$10 \in A^C$
T	F	$2 \subseteq A$		T	F	$\{\emptyset\} = \emptyset$		T	F	$0 = \emptyset$

17. Given two subsets A and B of U where $n(U) = 85$, $n(A \cup B) = 50$, $n(A) = 30$, and $n(B) = 45$, find $n(A \cup B^C)$.

18. $A = \{k, l, b\}$

(a) How many total subsets does the set A have?

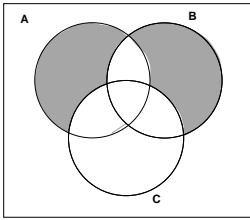
(b) List all the subsets of A .

(c) How many of these subsets are proper subsets of A ?

(d) Give an example of two subsets of A that are disjoint. If this is not possible, then explain why not.

19. Shade the part of a Venn diagram that is represented by $(A^C \cup B) \cap (C \cup A)$

20. Write the set notation that would represent the shaded portion of the Venn diagram.



21. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 4, 7, 8\}$, and $C = \{2, 4, 6, 8\}$.
Compute the following.

(a) $(A \cap B) \cup C$

(b) $A^C \cap B$

(c) $A \cap (B \cup C)^C$

22. 200 students were surveyed about their regular vending machine purchases. 150 regularly purchase drinks, 75 regularly purchase snacks, and 30 don't make regular vending machine purchases. How many of the surveyed students regularly

(a) Purchase both drinks and snacks?

(b) Purchase only drinks?

(c) Do not purchase drinks?

23. In a survey of 300 high school seniors:

- 120 had read *As You Like It* or *Romeo and Juliet*, but had not read *Macbeth*.
- 61 had read *As You Like It* but not *Romeo and Juliet*.
- 15 had read *Macbeth* and *As You Like It*.
- 14 had read *As You Like It* and *Romeo and Juliet*.
- 9 had read *Macbeth* and *Romeo and Juliet*.
- 5 had read *Macbeth* and *Romeo and Juliet*, but not *As You Like It*.
- 40 had read only *Macbeth*.

Let $M = \text{Macbeth}$, $R = \text{Romeo and Juliet}$, and $A = \text{As You Like It}$.

- (a) Fill in a Venn diagram illustrating the above information.
- (b) How many students read exactly one of these books?
- (c) How many students did not read *Romeo and Juliet*?
- (d) How many students read *Romeo and Juliet* and also had read *Macbeth* or *As You Like It*?
- (e) Compute the value of $n(M \cup (R^C \cap A))$ and write a sentence describing this information in words.
- (f) Compute the value of $n(A^C \cap (R \cup M))$ and write a sentence describing this information in words.