Week-In-Review #7 (7.4, 7.5, 7.6) \( p(E) = 1 - p(E^c) \)

1. A bag contains 4 red disks, 7 green disks and 8 blue disks. Suppose a sample of 6 disks are selected at random. What is the probability that the sample contains

(a) exactly 3 red and 3 blue disks?

\[
\frac{\binom{4}{3} \cdot \binom{8}{3}}{\binom{19}{6}} = \frac{8}{969}
\]

(b) all the available red disks?

\[
\frac{\binom{4}{4} \cdot \binom{15}{2}}{\binom{19}{6}} = \frac{5}{1292}
\]

(c) at least 2 blue disks?

\[
P(\text{at least 2B}) = 1 - P(\text{less than 2 blue})
\]

\[
= 1 - \left[ \frac{\binom{8}{0} \cdot \binom{11}{6} + \binom{8}{1} \cdot \binom{11}{5}}{\binom{19}{6}} \right]
\]

\[
= \frac{547}{646}
\]
(d) all disks of the same color?

\[
\frac{C(7,6) + C(8,6)}{C(19,6)} = \frac{5}{3876}
\]

(e) exactly 2 green or exactly 3 blue disks?

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B)
\]

\[
\frac{C(7,2)C(12,4) + C(8,3)C(11,3) - C(7,2)C(8,3)c(4,1)}{C(19,6)} = \frac{711}{1292}
\]
2. You roll a fair six-sided die 10 times. Each time a 5 is rolled you receive $5. What's the probability that you receive at least $40?

At least $\$40 \Rightarrow \$40 \text{ or } \$45 \text{ or } \$50$

\[ \text{8} \text{fives} \quad \text{9} \text{fives} \quad \text{10} \text{fives} \]

\[ \binom{10}{8} \cdot 1^8 \cdot \binom{2}{2} \cdot 5^2 + \binom{10}{9} \cdot 1^9 \cdot \binom{1}{1} \cdot 5 + \binom{10}{10} \cdot 1^{10} \]

\[ = \frac{1176}{60,469} \]

Example:

1 1 5 5 1 1 5 5 1 1

Total possible:

\[ b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = 6^{10} \]
Roll one die 3 times

\[ P(\text{at least 2 fives}) \Rightarrow \begin{array}{c}
2\text{ fives} \\
1\text{ five} \\
3\text{ fives}
\end{array} \]

\[ \frac{\binom{3}{2} \cdot 1^2 \cdot \binom{1}{1} \cdot 5 + \binom{3}{3} \cdot 1^3}{6^3} \]

Exactly 2 fives:

\[ \left( \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{5}{\text{NF}} \right) + \left( \frac{5}{\text{NF}} \cdot \frac{1}{5} \cdot \frac{1}{5} \right) + \left( \frac{1}{5} \cdot \frac{5}{\text{NF}} \cdot \frac{1}{5} \right) \]

\[ = \binom{3}{2} \cdot 1^2 \cdot \binom{1}{1} \cdot 5 \]

Exactly 3 fives:

\[ \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} = \binom{3}{3} \cdot 1^3 \]

Total possible:

\[ b \cdot b \cdot b = b^3 \]
3. You roll two fair five-sided dice (one red and one green). What is the probability that

\[ 5 \cdot 5 = 25 \]

outcomes

(a) the sum is 7?

\[ P(\text{Sum} = 7) = \frac{4}{25} \]

(b) the red die shows a bigger number than the green die?

\[ P(R > G) = \frac{10}{25} \]

(c) the sum of the dice is odd and the green die shows a 2?

\[ P(\text{Sum} = \text{odd} \cap G = 2) = \frac{3}{25} \]

(d) the red die shows a 2 or the green die shows a 4?

\[ P(R = 2 \cup G = 4) = \frac{9}{25} \]

(e) the sum is odd if the green die shows a 2?

\[ P(\text{Sum odd} \mid G = 2) = \frac{3}{5} \]

\[ P(R = 2 \cup G = 4) = P(R = 2) + P(G = 4) - P(R = 2 \cap G = 4) \]

\[ = \frac{5}{25} + \frac{5}{25} - \frac{1}{25} \]

\[ P(\text{Sum odd} \cap G = 2) = \frac{P(\text{Sum odd} \cap G = 2)}{P(G = 2)} \]

\[ = \frac{\frac{3}{5}}{\frac{3}{5}} = \frac{3}{5} \]
4. In order to use an ATM machine, you must have a 4-digit pin number. Suppose that to pick your pin, you number pieces of paper from 0 through 9 and place them in a bag. You then draw 4 pieces of paper from the bag, in succession, with replacement. The first number drawn becomes the first digit of your pin the second number drawn becomes the second digit, etc. What is the probability that your pin number will have exactly three 2's in it?

\[
\frac{\binom{4}{3} \cdot 1^3 \cdot \binom{1}{1} \cdot 9}{10^4} = \frac{9}{2500}
\]

Total # possible: \[10 \cdot 10 \cdot 10 \cdot 10 = 10^4\]
5. For Sunday dinner, the Smith family and the Jones family go out to eat. Twenty percent of the time, the Smiths go to the Elite Diner. Thirty percent of the time, the Joneses go to the Elite Diner. Twelve percent of the time, both families go to the Elite Diner.

Let $S$ and $J$ represent the obvious events. Sketch an appropriate Venn diagram. Use it to answer the following questions.

(a) What is the probability that on any given Sunday,

(i) at least one of the families goes to the Elite Diner?

$$ P(S \cup J) = a + b + c = 0.38 $$

(ii) exactly one of the families goes to the Elite Diner?

$$ P[(S \cap J^c) \cup (J \cap S^c)] = a + c = 0.26 $$

(iii) neither of the two families goes to the Elite Diner?

$$ P(S^c \cap J^c) = d = 0.62 $$

(iv) the Joneses will eat at the Elite Diner, if it is known that the Smith family is eating at the diner?

$$ P(J \mid S) = \frac{P(J \cap S)}{P(S)} = \frac{b}{a + b} = \frac{0.12}{0.2} = 0.6 $$

(b) Are $S$ and $J$ mutually exclusive events? Why or why not?

**No,** $P(S \cap J) \neq 0$ (Jones and Smiths can both eat at diner at same time)

(c) Are $S$ and $J$ independent events? Why or why not?

**Independent:** $P(S \cap J) = P(S) P(J)$

$$ \frac{0.12}{0.2} \neq \frac{0.2}{0.2} \times \frac{0.3}{0.2} $$

**Dependent** $\not\Rightarrow$ **Not Independent** $\not\Rightarrow 0.12 \neq 0.6$.
6. A survey of 5902 adults revealed the following:

<table>
<thead>
<tr>
<th></th>
<th>Has seen ghost</th>
<th>Has not seen ghost</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y= Age 18-29</td>
<td>212</td>
<td>1313</td>
<td>1525</td>
</tr>
<tr>
<td>Y= Age 30 or over</td>
<td>465</td>
<td>3912</td>
<td>4377</td>
</tr>
<tr>
<td>Total</td>
<td>677</td>
<td>5225</td>
<td>5902</td>
</tr>
</tbody>
</table>

Let $Y = \text{adult is under 30}$, $V = \text{adult is 30 or over}$, $G = \text{adult has seen a ghost}$, $N = \text{adult has not seen a ghost}$

(a) What is the probability that a randomly selected adult has reportedly seen a ghost?

$$P(G) = \frac{n(G)}{n(S)} = \frac{677}{5902}$$

(b) What is the probability that a randomly selected adult is under 30 and has seen a ghost?

$$P(Y \cap G) = \frac{n(Y \cap G)}{n(S)} = \frac{212}{5902}$$

(c) What is the probability that a randomly selected young adult has seen a ghost?

$$P(G|Y) = \frac{P(G \cap Y)}{P(Y)} = \frac{212}{1525} \div \frac{677}{5902}$$

(d) What is the probability that an adult who has seen a ghost is under 30?

$$P(Y|G) = \frac{212}{677}$$

(e) What is the probability that a randomly selected adult is under 30 or has seen a ghost?

$$P(Y \cup G) = \frac{1990}{5902}$$

\[
P(Y \cup G) = P(Y) + P(G) - P(Y \cap G) = \frac{1525}{5902} + \frac{677}{5902} - \frac{212}{5902}
\]

(f) Are the events, "adult is under 30" and "adult has seen a ghost", independent or dependent?

$$P(Y \cap G) \overset{?}{=} \frac{212}{5902} \overset{?}{=} \left( \frac{1525}{5902} \right) \left( \frac{677}{5902} \right)$$

Dependent
7. There are 10 people in a class who each perform independently from one another on a test. If five people in the class each have an 80% chance of passing a test and the other five each have a 30% chance of passing the test, what's the probability that everyone in the class passes the test? 

\[
P(\cap_{i=1}^{10} p_i) = P(p_1)P(p_2)P(p_3)\ldots P(p_{10})
\]

\[
= (0.8)(0.8)(0.8)(0.8)(0.8)(0.3)(0.3)(0.3)(0.3)(0.3)
\]

\[
= (0.8)^5(0.3)^5
\]

\[
\approx 7.9426 \times 10^{-4} \approx 0.00079626
\]
Fill in the missing probabilities in the tree above and then answer the following:

(a) $P(A^C) = 1 - P(A) = 1 - 0.2 = \boxed{0.8}$

(b) $P(D \mid C) = \boxed{0.8}$

(c) $P(C \cap D) = P(C)P(D \mid C) = 0.4 \times 0.8 = \boxed{0.32}$ \hspace{1cm} (path on tree)

(d) $P(E) = P(A \cap E) + P(B \cap E) + P(C \cap E) = (0.2)(0.9) + (0.4)(0.75) + (0.4)(0.2) = \boxed{0.56}$ \hspace{1cm} (all paths ending at $E$)

(e) $P(E^C) = 1 - P(E) = 1 - 0.56 = \boxed{0.44}$
8. (cont')

\[ P(D^c) = 1 - P(D) = 1 - [0.44] = 0.56 \]

\( P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{0.4 \cdot 0.8}{0.2 \cdot 0.1 + 0.4 \cdot 0.25 + 0.4 \cdot 0.8} = \frac{0.32}{0.44} = \frac{8}{11} \)

\( P(B \cup D) = P(B) + P(D) - P(B \cap D) = 0.4 + \boxed{[0.44]} - 0.4 \cdot 0.25 = 0.74 \)

\( P(B \cap D^c) = P(B \cap E) = 0.4 \cdot 0.75 = 0.3 \)

\( P(A \cup D^c) = P(A) + P(D^c) - P(A \cap D^c) = 0.2 + \boxed{0.56} - 0.2 \cdot 0.9 = 0.58 \)

(j) Are A and E mutually exclusive? Independent?

M.E.: No \(\Rightarrow P(A \cap E) = 0.2 \cdot 0.9 \neq 0 \) \(\Rightarrow\) Path containing both A and E

Indep.: \(\frac{P(A \cap E)}{P(A) \cdot P(E)} = \frac{0.2 \cdot 0.9}{0.2 \cdot [0.56]} \neq 0.2 \Rightarrow\) Not Indep. \(\Rightarrow\) Dependent
9. On Tuesday night, in a group of people, 60% watch shows on NBC and 40% watch shows on CBS. Of those who watch NBC, 50% watch TV at 7:00 PM and of those who watch CBS, 80% watch TV at 7:00PM.

(a) Draw a tree representing the situation.

(b) What’s the probability that a randomly selected person from the group watches CBS on Tuesday night?

\[ P(C) = 0.4 \]

(c) What’s the probability that a randomly selected person watches CBS on Tuesday and watches TV at 7:00PM?

\[ P(C \cap S) = 0.4 \times 0.8 = 0.32 \]

(d) What’s the probability that a randomly selected person watches TV at 7:00PM on Tuesday?

\[ P(S) = 0.6 \times 0.5 + 0.4 \times 0.8 = 0.62 \]

(e) If you know a person from the group watches TV at 7:00PM on Tuesday, is it more likely they are watching CBS or NBC?

\[ P(N|S) = \frac{P(N \cap S)}{P(S)} = \frac{0.6 \times 0.5}{0.62} = \frac{15}{31} \]

\[ P(C|S) = \frac{P(C \cap S)}{P(S)} = \frac{0.4 \times 0.8}{0.62} = \frac{16}{31} \]
10. Draw two cards in succession, without replacement, from a standard 52 card deck.

\[ A_1 = \text{ace on 1st} \]
\[ A_2 = \text{ace on 2nd} \]

(a) Given that the first card drawn is an Ace, what is the probability that the second card drawn is an Ace?

\[ P(A_2 | A_1^c) = \frac{3}{51} \]

(b) What is the probability that the second card drawn is an Ace?

\[ P(A_2) = \frac{4}{52} \left( \frac{3}{51} \right) + \frac{48}{52} \left( \frac{4}{51} \right) = \frac{1}{13} \]

(c) Given that the second card drawn is an Ace, what is the probability that the first card drawn is an Ace?

\[ P(A_1 | A_2^c) = \frac{P(A_1 \cap A_2)}{P(A_2)} = \frac{\frac{4}{52} \left( \frac{3}{51} \right)}{\frac{1}{13}} = \frac{1}{17} \]
11. Consider the following experiment: Select a ball at random from a bowl (which has 4 red and 6 green balls) and place it into a bag (which has 7 red and 2 green balls). Then, draw a ball at random from the bag.

(a) Sketch an appropriate tree diagram for this experiment.

(b) What is the probability that both balls drawn are red?

\[ P(R_1 \cap R_2) = \left(\frac{4}{10}\right) \left(\frac{8}{10}\right) = 0.32 \]

(c) What is the probability that the second ball drawn is red?

\[ P(R_2) = \frac{4}{10} \left(\frac{8}{10}\right) + \frac{6}{10} \left(\frac{7}{10}\right) = 0.74 \]

(d) If the second ball drawn is red, what is the probability that the first ball drawn is red?

\[ P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{0.32}{0.74} = \frac{16}{37} \]

(e) What is the probability that a green was drawn from the bowl or from the bag?

\[ P(G_1 \cup G_2) = P(G_1) + P(G_2) - P(G_1 \cap G_2) = \left(\frac{6}{10}\right) + \left[\frac{4}{10} \left(\frac{7}{10}\right) + \frac{6}{10} \left(\frac{3}{10}\right)\right] - \left(\frac{1}{10}\right) \left(\frac{3}{10}\right) = 0.68 \]
12. A test for "cooties" correctly indicates the presence of the condition 99% of the time among those who actually have "cooties". However, 5% of the time, the test indicates the presence of the condition in those who do not actually have it. It is known that 8% of those who take the test actually have "cooties".

\[ D = \text{has cooties} \]
\[ +/\sim = 1^{st} \text{ test results} \]
\[ p/n = 2^{nd} \sim \]

(a) If the test given to a person comes back positive, what is the probability that the person actually has "cooties"?

\[ P(D \mid +) = \frac{P(D \cap +)}{P(+)} = \frac{0.08(0.99)}{0.08(0.99) + 0.92(0.05)} = \frac{198}{313} \approx 0.6326 \]

(b) What is the probability that a person actually has "cooties", given that they take the test twice and both times the test is positive for the condition?

\[ P(D \cap + \cap p) = \frac{P(D \cap + \cap p)}{P(+ \cap p)} = \frac{0.08(0.99)(0.99)}{0.09(0.99)(0.99) + 0.92(0.05)(0.05)} = \frac{0.9715}{0.9715} = 1 \]