Week-In-Review #8 (8.1, 8.2, 8.3)

1. Classify the following random variables as finite discrete, infinite discrete, or continuous AND give all possible values the random variable may take.

   - Infinite Discrete: Pattern to the values, but no endpoint
   - Continuous: Values comprise an interval of real #'s

(a) \( X = \) the number of times you blow your nose when you're sick

   Infinite Discrete \( X = 0, 1, 2, 3, \ldots \)

(b) \( X = \) the number of times you draw a card from a standard 52-card deck (without replacement), if you draw a card until you get a 4

   Finite Discrete \( X = 1, 2, 3, \ldots, 49 \)

(c) \( X = \) the number of minutes you pay attention in a MWF math class (50 min long)

   Continuous \( 0 \leq X \leq 50 \)
(d) You have a bag with 1 green, 2 blue and 5 yellow marbles.

(i) You choose a marble, with replacement, until the green marble is drawn.

\[ X = \text{the number of times you draw a marble} \]

\[ \text{Infinite Discrete} \quad X = 1, 2, 3, \ldots \]

(ii) You choose a sample of 6 marbles from the bag.

\[ X = \text{the number of yellow marbles in your sample} \]

\[ \text{Finite Discrete} \quad X = 3, 4, 5 \]

(e) \( X = \text{the number of ounces of Diet Dr Pepper you drink in one day} \)

\[ \text{Continuous} \quad X \geq 0 \]

(f) \( X = \text{the number of hours of TV you watch, to the nearest half hour, in one day} \)

\[ \text{Finite Discrete} \quad X = 0, \frac{1}{2}, 1, 1.5, 2, \ldots, 24 \]
2. 

(a) Find the value of $P(X = 3) = ? = 1 - 0.82 \pm 0.18$

(b) Find

(i) $P(X \geq 1) = p(X=1) + p(X=2) + p(X=3) = 0.02 + 0.5 + 0.18 = 0.7$

(ii) $P(-2 < X < 5) = 1 - p(X=-2) = 1 - 0.1 = 0.9$

(iii) $P(X = 2.5) = 0$

(c) Draw a histogram representing the given information.

(d) Find the following values for $X$:

(i) Expected Value $E(X) = \bar{x} = 1.2$

(ii) Median $\text{Med} = 2$

(iii) Mode $2$

(iv) Standard Deviation $\sigma_X = 1.6510$

(v) Variance $(\sigma_X)^2 = 2.7259$
3. A group of movie buffs was surveyed about the number of Tom Cruise movies they own and the following was found. Place an X next to the row of the table which would represent the random variable for this data and answer the questions which are listed below.

\[
\begin{array}{c|ccccc}
\text{F} & \# \text{ of movie buffs} & 5 & 6 & 3 & 20 & 2 \\
\text{X} & \# \text{ of movies} & 1 & 2 & 3 & 4 & 5 \\
\end{array} \quad \text{Sum} = 36
\]

(a) Find the mean, median, mode, variance, and standard deviation for the number of Tom Cruise movies owned per person.

\[
\text{Mean} = \bar{x} = 3.2222 \Rightarrow \frac{29}{9} \text{ movies}
\]

\[
\text{Median} = \text{Med} = 4 \text{ movies}
\]

\[
\text{Mode} = 4 \text{ movies}
\]

\[
\text{SD} = \sigma_X = 1.2044 \text{ movies}
\]

\[
\text{Variance} = (\sigma_X)^2 = \frac{235}{162} \approx 1.4506 \text{ (movies)}^2
\]

(b) What is the probability that a randomly selected movie buff owns at least two Tom Cruise movies?

\[
p(X \geq 2) = 1 - p(X = 1)
\]

\[
= 1 - \left( \frac{5}{36} \right) = \frac{31}{36}
\]
4. In a bag of 15 Clementine oranges, 3 are bruised. You choose a sample of 4 oranges at random from the bag. Let $X$ represent the number of bruised oranges in your sample.

(a) Find the probability distribution of $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>$\frac{C(3,0) \cdot C(12,4)}{C(15,4)}$</td>
<td>$\frac{C(3,1) \cdot C(12,3)}{C(15,4)}$</td>
<td>$\frac{C(3,2) \cdot C(12,2)}{C(15,4)}$</td>
<td>$\frac{C(3,3) \cdot C(12,1)}{C(15,4)}$</td>
</tr>
<tr>
<td>$n$</td>
<td>33/31</td>
<td>44/91</td>
<td>66/455</td>
<td>4/455</td>
</tr>
</tbody>
</table>

(b) Find $P(X < 2) = P(X=0) + P(X=1)$

$$= \frac{33}{91} + \frac{44}{91} = \frac{77}{91} = \frac{11}{13}$$

(c) How many bruised oranges would you expect to find in your sample?

$$E(X) = \frac{22}{91} + 1 \left( \frac{44}{91} \right) + 2 \left( \frac{66}{455} \right) + 3 \left( \frac{4}{455} \right)$$

$$= 0.8 \quad \rightarrow \text{Realistically, you can expect 0 or 1 bruised orange}$$
5. A car insurance policy covers damages from a car accident. According to your policy, if you get into a "fender-bender", the insurance company will pay out $300, and if you get into a major accident, the company will pay out $2000.

(a) Suppose your monthly payment is $150, the probability that you get into a "fender-bender" is 0.2, and the probability that you get into a major accident is 0.02. What is the insurance company's expected gain?

\[ E(X) = (-150)(0.2) + (-1850)(0.02) + (150)(0.78) = 450 \]

(b) After getting in several accidents, you provide an extra risk to the insurance company, as they have determined your probability of getting into a "fender bender" to be 0.5 and that of getting into a major accident is 0.08. If the insurance company requires an expected gain that is greater than or equal to $0, what would your new minimum monthly payment be?

\[ E(X) = (m-300)(0.5) + (m-2000)(0.08) + m(0.42) = 0 \]

\[ 0.5m - 150 + 0.8m - 160 + 0.42m = 0 \]

\[ 2.72m = 310 \]

\[ m = \$310 \]
6. You pay $2.00 to play in a game where you roll a fair die and toss two fair coins. If both coins come up heads, you win twice the amount shown on the die in dollars. If both coins come up tails, you win the amount shown on the die in dollars. Otherwise, you win nothing.

(a) Draw a probability distribution describing your NET winnings.

(b) What are your expected net winnings?

\[ E(X) = (-2) \left( \frac{1}{24} \right) + (-1) \left( \frac{1}{24} \right) + \cdots + 10 \left( \frac{1}{24} \right) \]

\[ = \frac{15}{24} = 0.625 \rightarrow \text{Expected about } 63 \text{ } \]

(c) Is this a fair game? Why or why not?

No \textbf{no} expected net winnings \textbf{not} winnings \textbf{not} 0.
7. You enter a game where you draw one letter from a bag containing a tile for each letter in the word BULLABALOO. You win $10 if you choose an “L”, $1 if you choose a vowel, and nothing if you choose any other letter. How much should you expect to pay in order to make this a fair game?

\[ E(X) = 0 \rightarrow X = \text{net winnings} \]

<table>
<thead>
<tr>
<th>Outcome</th>
<th>&quot;L&quot;</th>
<th>Vowel</th>
<th>Choose something else</th>
</tr>
</thead>
<tbody>
<tr>
<td>X = net win</td>
<td>10 - p</td>
<td>1 - p</td>
<td>0 - p</td>
</tr>
<tr>
<td>prob</td>
<td>( \frac{3}{10} )</td>
<td>( \frac{5}{10} )</td>
<td>( \frac{2}{10} )</td>
</tr>
</tbody>
</table>

\[ E(X) = (10 - p) \left( \frac{3}{10} \right) + (1 - p) \left( \frac{5}{10} \right) + (0 - p) \left( \frac{2}{10} \right) = 0 \]

\[ 3 - 0.3p + 0.5 - 0.5p - 0.2p = 0 \]

\[ 3.5 - p = 0 \]

\[ p = \$3.50 \]
8. The probability that John will go to class tomorrow is 0.95.

(a) What are the odds that John will go to class tomorrow?

\[
\begin{align*}
\text{P(John goes to class)} &= 0.95 \\
\text{P(John doesn't go)} &= 0.05
\end{align*}
\]

\[
\frac{0.95}{0.05} = 19 = \frac{19}{1} \Rightarrow 19 \text{ to } 1 \text{ or } 19:1
\]

(b) What are the odds that John won't go to class tomorrow?

\[
\begin{align*}
\text{P(doesn't)} \rightarrow \frac{0.05}{0.95} &= \frac{1}{19} \Rightarrow 1 \text{ to } 19 \text{ or } 1:19
\end{align*}
\]
9. You read in the paper that a particular horse has 13 to 3 odds of winning its next race.

(a) What is the probability that the horse will win its next race?

\[ P(\text{win}) = \frac{n(\text{win})}{n(\text{total races})} = \frac{13}{13+3} = \frac{13}{16} \]

(b) What is the probability that the horse will not win its next race?

\[ P(\text{not win}) = \frac{n(\text{not win})}{n(\text{total races})} = \frac{3}{16} \]
10. You roll two fair six-sided dice.

(a) Draw the probability distribution associated with the random variable, $X$, which denotes the sum of the numbers rolled.

<table>
<thead>
<tr>
<th>X: Sum</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
</table>

(b) What sum do you expect to roll?

$$E(X) = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + \ldots + 12\left(\frac{1}{36}\right) = 7$$

(c) What are the odds that you roll a sum greater than 9?

$$P(\text{Sum} > 9) = \frac{6}{36}$$
$$P\left(\text{Sum} \leq 9\right) = \frac{30}{36} = \frac{5}{6}$$