Week-In-Review #10 (5.1, 5.2, 5.3)

1. Find the amount of money at the end of 5 years on a $200 deposit in an account paying simple interest at a rate of 4.75% per year. How much interest is earned?

   \[
   A = P(1 + rt) = 200(1 + 0.0475 \times 5) = 247.50
   \]
   \[
   I = A - P = 247.50 - 200 = 47.50
   \]

2. A bank deposit paying simple interest grew from an initial sum of $2000 to a sum of $2150 in 7 months. Find the interest rate.

   \[
   I = P \times r \times t = 2150 - 2000 = 150
   \]
   \[
   150 = 2000 \times r \times \frac{7}{12}
   \]
   \[
   r = \frac{150 \times 12}{2000 \times 7} = \frac{18}{7} \approx 2.57 \%
   \]

3. How much money will be in an account after 2 years on a $500 deposit that earns interest at a rate of 5% per year compounded continuously?

   \[
   A = 500e^{0.05 \times 2} = 552.59
   \]
   \[
   I = A - P = 552.59 - 500 = 52.59
   \]
4. If Mark invests $5000 into an account paying interest at a rate of 8% per year compounded monthly, how much money will he have at the end of 5 years (assuming no additional deposits or withdrawals)?

\[
\begin{align*}
TVM_{\text{Solver}} \\
N &= 12 \times 5 \\
I &= 8 \\
PV &= -5000 \\
PMT &= 0 \\
FV &= ? \\
p/y &= c/y = 12 \\
&\rightarrow $7449.23 \\
\end{align*}
\]

Interest: $7449.23

\[
\begin{align*}
&-5000 \\
&\underline{\underline{2449.23}} \\
\end{align*}
\]

5. In 18 months Brian needs $1750 in order to buy a specific computer. If he finds an account paying interest at a rate of 5.95% per year compounded weekly, how much could he invest now in order to have the money he needs for the computer?

\[
\begin{align*}
TVM_{\text{Solver}} \\
N &= 52 \times 1.5 \\
I &= 5.95 \\
FV &= ? \\
PMT &= 0 \\
PV &= 1750 \\
p/y &= c/y = 52 \\
&\rightarrow $1600.66 \\
\end{align*}
\]

\[
\begin{align*}
t &= \frac{18 \text{ mo}}{1 \text{ yr}} = 1.5 \text{ yr} \\
12.0\% \\
\end{align*}
\]
TVM Solver

N = total # of compounding periods = mt
I % = interest rate as a %

PV = present value
PMT = payment amount
FV = future value

\( p/y = \# \text{ of payments per year} = m \)
\( c/y = \# \text{ of compd. pds per year} = m \)

| END |

To solve: Use ALPHA + ENTER

m = # of compd. pds per year
\( t = \# \text{ of years} \)
6. Kevin inherits $50,000. If he invests it by placing it into an account paying interest at a rate of 10.5% per year compounded monthly, how long would he have to leave his money in the account before having $100,000?

\[ N = \frac{m \cdot t}{12} \]

\[ m = 12 \]

\[ N = ? \]

\[ I = 10.5 \]

\[ PV = -50000 \]

\[ PMT = 0 \]

\[ FV = 100000 \]

\[ P/Y = C/Y = 12 \]

\[ \Rightarrow 79.562 \ldots \text{months} \]

\[ \Rightarrow \frac{79.562}{12} \approx 6 \text{ yrs and 8 mo.} \]

7. What interest rate, compounded daily, will quadruple $1200 after 5 years?

\[ m = 365 \]

\[ N = 365 \times 5 \]

\[ PV = -1200 \]

\[ PMT = 0 \]

\[ FV = 4800 \]

\[ P/Y = C/Y = 365 \]

\[ \Rightarrow I \approx 27.74\% \text{ per year} \]

8. A major credit card company has a finance charge of 1.5% per month on the outstanding indebtedness. Susie charged $1000 and did not pay her bill for 6 months. What is the bill after the 6 months?

\[ \frac{1.5\%}{12 \text{ mo}} \times 12 \text{ yr} = 18\% \text{ yr} \]

\[ N = 12 \times \frac{1}{2} = 6 \]

\[ I = 18 \]

\[ FV = ? \]

\[ PV = 1000 \]

\[ P/Y = C/Y = 12 \]

\[ \Rightarrow \#1093.44 \]
9. Which account would be a better account for an investment?

- OPTION A: 9% per year, compounded monthly
- OPTION B: 8.8% per year, compounded daily
- OPTION C: 8.9% per year, compounded continuously

\[
\text{EFF}(r, m) = \left(1 + \frac{r}{m}\right)^m - 1
\]

\[
\text{EFF}(9, 12) \approx 9.38\%
\]

\[
\text{EFF}(8.8, 365) \approx 9.20\%
\]

10. If Louis deposits $50 at the end of each month into a savings account earning interest at a rate of 7.25% per year compounded monthly, how much will he have at the end of 30 years (assuming that he makes no withdrawals during that period)? How much interest will he earn?

\[\text{TVMSolver} \quad \begin{align*}
N &= 12 \times 30 \\
I &= 7.25 \\
P &= 0 \\
P+i &= -50 \\
F &= ? \\
\Pi &= 12 \\
\Pi = \Pi \\
\end{align*}\]

\[
\text{# deposited: } 50 \times (12)(30) = 18000
\]

\[
\text{Interest} = 46094.66 - 18000 = 46094.66
\]
11. You are looking to buy a new car and have only $1000 for a down payment. The car you wish to buy has a cash price of $22,500. If the best financing option you find charges interest at a rate of 3.5% per year compounded monthly, how big would your monthly payments be in order to pay off the car in 60 months, assuming you use the money you already have for a down payment? In 48 months? In 36 months?

\[
\text{Cash Price} = \text{Down Pmt} + \text{Loan Amt}
\]
\[
22500 = 1000 + \text{Loan Amt}
\]
\[
\text{Loan Amt} = 21500
\]

<table>
<thead>
<tr>
<th>60 mo. loan</th>
<th>48 mo. loan</th>
<th>36 mo. loan</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TVMSolver</strong></td>
<td><strong>TVMSolver</strong></td>
<td><strong>TVMSolver</strong></td>
</tr>
<tr>
<td>( N = 60 )</td>
<td>( N = 48 )</td>
<td>( N = 36 )</td>
</tr>
<tr>
<td>( I = 3.5 )</td>
<td>( I = 3.5 )</td>
<td>( I = 3.5 )</td>
</tr>
<tr>
<td>( PV = 21500 )</td>
<td>( PV = 21500 )</td>
<td>( PV = 21500 )</td>
</tr>
<tr>
<td>( PMT = ? )</td>
<td>( PMT = ? )</td>
<td>( PMT = ? )</td>
</tr>
<tr>
<td><strong>( FV = 0 )</strong></td>
<td><strong>( FV = 0 )</strong></td>
<td><strong>( FV = 0 )</strong></td>
</tr>
<tr>
<td>( P/Y = C/Y = 12 )</td>
<td>( P/Y = C/Y = 12 )</td>
<td>( P/Y = C/Y = 12 )</td>
</tr>
</tbody>
</table>

\( PMT = \$391.12 \) \( PMT = \$480.65 \) \( PMT = \$629.99 \)
12. Betsy decided to purchase a car. She makes a down payment of $4000 and secures financing for the balance of the purchase price at a rate of 4% per year compounded monthly. Under the terms of the finance agreement she is required to make payments of $300 each month for 48 months.

(a) What is the cash price of the car?

\[
\begin{align*}
\text{TVM Solver} \\
N &= 48 \\
I &= 4 \\
(PV, \text{Loan Amt}) &= ? \\
\text{PMT} &= -300 \\
FV &= 0 \\
\text{Ply, Cy} &= 12 \\
4000 + \text{Loan Amt} &= 17286.65 \\
\text{Loan Amt} &= 13286.65 \\
\end{align*}
\]

(b) How much interest does Betsy end up paying?

\[
\begin{align*}
\# \text{ pd in total} &= 300(48) = 14400 \\
-13286.65 \text{ loan a.m.t} \\
\hline
\# \text{ pd in interest} &= 1113.35 \\
\end{align*}
\]
13. A sum of $5000 is to be repaid over a 5 year period through equal installments made at the end of each year. If an interest rate of 6% per year is charged on the unpaid balance and interest calculations are made at the end of the year, determine the size of each installment so that the loan is amortized at the end of 5 years. Show the amortization schedule.

<table>
<thead>
<tr>
<th>End of Period</th>
<th>Pmts Remaining</th>
<th>Payment Pmts</th>
<th>TO Interest</th>
<th>TO Principal</th>
<th>Outstanding Principal</th>
<th>EQUITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
<td></td>
<td></td>
<td>5000 + 0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1186.98</td>
<td>300</td>
<td>886.98</td>
<td>4113.02 + 886.98</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1186.98</td>
<td>246.78</td>
<td>940.20</td>
<td>3172.82 + 940.20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1186.98</td>
<td>190.37</td>
<td>996.61</td>
<td>2176.21 + 996.61</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1186.98</td>
<td>130.57</td>
<td>1056.41</td>
<td>1119.80 + 1056.41</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1186.98</td>
<td>77.19</td>
<td>1119.79</td>
<td>0.01 + 1119.79</td>
<td></td>
</tr>
</tbody>
</table>

Payment Amt = Amt Twds Interest + Amt Twds Principal
pays down loan

6% of outstanding principal at end of previous period.
14. Sally buys a new $1000 television by paying $100 and financing the remaining $900. The terms of her finance agreement state that the unpaid balance will be charged interest at a rate of 15% per year, compounded monthly, and the money is to be repaid over a 2 year period through equal installments made at the end of each month.

(a) What will Sally's monthly payments be?

\[ \frac{PV}{PMT} \]
\[
\begin{align*}
N &= 12 \times 2 = 24 \\
I &= 15 \\
PV &= 900 \\
PMT &= ? \Rightarrow 43.64 \\
FV &= 0 \\
P/Y &= 12 \\
\end{align*}
\]

(b) How much of the first payment goes towards paying down the loan?

\[ PMT = Amt \ to \ Interest + Amt \ Towards \ Paying \ Loan \]
\[
\begin{align*}
43.64 &= 11.25 + ? \\
\Rightarrow \ Amt \ Towards \ Loan &= \frac{43.64}{11.25} \Rightarrow \ \frac{43.64}{32.39} \\
\end{align*}
\]

Mo. Int. Rate = \[ \frac{15\%}{12 \text{mo}} = 1.25\% / \text{mo} \]

\[ \text{Interest} = 900(0.0125) = 11.25 \]
(c) Fill in the first 6 lines of the amortization schedule.

<table>
<thead>
<tr>
<th>End of Period</th>
<th>Pmts Remaining</th>
<th>Payment</th>
<th>TO Interest</th>
<th>TO Principal</th>
<th>Outstanding Principal</th>
<th>EQUITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
<td>0</td>
<td></td>
<td></td>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>23</td>
<td>43.64</td>
<td>11.25</td>
<td>32.39</td>
<td>867.61</td>
<td>132.39</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>43.64</td>
<td>10.85</td>
<td>32.79</td>
<td>834.82</td>
<td>165.18</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>43.64</td>
<td>10.44</td>
<td>33.20</td>
<td>801.62</td>
<td>198.38</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>43.64</td>
<td>10.02</td>
<td>33.62</td>
<td>768</td>
<td>232</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
<td>43.64</td>
<td>9.60</td>
<td>34.04</td>
<td>733.96</td>
<td>266.04</td>
</tr>
</tbody>
</table>

(d) How much equity will Sally have in her computer after 1 year?

TVSOLVE
N = 12 (\# of payments remaining)
I = 15%
FV = ?
PMT = -43.64
FV = 0
P/Y = C/Y = 12

\[ \text{EQUITY} = \frac{\text{Total Worth} - \text{Amnt we Owe}}{\text{after 12 payments}} \]

\[ \text{EQUITY} = \frac{1000 - 483.50}{12} \]

\[ \text{EQUITY} = \$516.50 \]
15. Ten years ago Quincy made a down payment on a house of 20% of the purchase price and secured a bank loan of $72,500 to finance the remaining amount. The mortgage was for a term of 30 years, with an interest rate of 7.25% per year compounded monthly on the unpaid balance to be amortized through equal monthly payments.

(a) What is the outstanding principal on Quincy’s house now?

\[
\begin{align*}
N &= 12 \times 30 = 360 \\
I &= 7.25 \\
\text{PV} &= ? \\
\text{PMT} &= -494.58 \\
FV &= 0 \\
\text{FV} &= 0 \\
\text{PMT} &= ? \\
\text{PV} &= 0 \\
\text{C/Y} &= 12 \\
\\text{Solution:}\ 
\text{PV} &= 62575.28 \\
\end{align*}
\]

(b) How much equity does Quincy have in the house now?

\[
\begin{align*}
\text{EQ} &= \text{Total Worth} - \text{Amount You Owe} \\
&= 90625 - 62575.28 \\
&= 28049.72
\end{align*}
\]

(c) How much total interest will Quincy pay over the life of the loan?

\[
\text{Pd in total} = (494.58)(360) = 178049.80
\]

\[
\text{Total Loan Amount} = 72500
\]

\[
\text{Total Interest} = 105,548.80
\]
(d) At this time, interest rates have dropped to 5% per year compounded monthly on a 15-year mortgage and Quincy is thinking about refinancing. If he refineses, what will his new payments be?

\[ \text{TVMSolver} \]

\[ N = 12 \times 15 = 180 \]
\[ I = 5 \]
\[ FV = 62575.28 \text{ (what remains on old loan)} \]
\[ P\text{M}\text{F} = ?? \rightarrow \$ 444.84 \]
\[ FV = 0 \]
\[ P / y = c / y = 12 \]

(e) How much money, if any, will Quincy save by refinancing (assuming no additional refinancing costs)?

No refinance (old loan) \[ \Rightarrow 20 \text{ yrs left} \]
\[ \Rightarrow 12 \times 20 = 240 \text{ pmts left of } \$ 494.58 \]
\[ \Rightarrow \$ 118699.20 \]

Refinance (new loan) \[ \Rightarrow 15 \text{ yrs left} \]
\[ \Rightarrow 12 \times 15 = 180 \text{ pmts left of } \$ 494.84 \]
\[ \Rightarrow \$ 89071.20 \]
\[ 118699.20 - 89071.20 = \$ 29628 \text{ saved} \]
16. You wish to retire with $1,000,000 in a retirement account, which you will make equal monthly deposits to during the 45 years that you work. If the account will pay interest at a rate of 5% per year compounded monthly, how much should you deposit each month in order to have your million?

\[
\text{TVM Solver}
\]
\[
N = 12 \times 45
\]
\[
I = 5
\]
\[
Pv = 0
\]
\[
Fv = 1,000,000
\]
\[
M/r = 12
\]
\[
Fv = \$ 493.48
\]

How much of the $1,000,000 did you deposit?

\[
(493.48)(12 \times 45) = \$ 266,479.20
\]

How much interest did you earn?

\[
1,000,000 - 266,479.20 = \$ 733,520.80
\]