Exam II Review Problems

Note: Not every topic is covered in this review.
Please also take a look at the previous Week-in-Reviews for more practice problems.

1. You have completely forgotten the combination of your lock. It is one of those “standard” combination locks, which uses a three number combination with each number in the range of 0 through 39. All you remember is that the second number is either 27 or 37, while the third number ends in a 5. In desperation, you decide to go through all possible combinations. Assuming that it takes about 10 seconds to try each combination, what is the longest possible time it can take to open your locker?

\[ 40 \times 2 \times 4 = 320 \text{ combinations} \times 10 \text{ sec} \]

\[ = 3200 \text{ sec} \]

\[ = 53\frac{1}{3} \text{ min} \]
2. Many U.S. license plates display a sequence of three letters followed by three digits.

(a) How many such license plates are possible? (only use cap. letters)

\[
\frac{26}{L} \times \frac{26}{L} \times \frac{26}{L} \times \frac{10}{D} \times \frac{10}{D} \times \frac{10}{D} = 17,576,000
\]

(b) In order to avoid confusion of letters with digits, some states do not use the letters I, O or Q on their license plates. How many of these license plates are possible?

\[
\frac{23}{L} \times \frac{23}{L} \times \frac{23}{L} \times \frac{10}{D} \times \frac{10}{D} \times \frac{10}{D} = 12,167,000
\]
3. Bill and Sue and four of their friends go to the movies. They all sit next to each other in the same row. How many ways can this be done if

(a) Sue and Bill must sit next to each other?

\[
\begin{align*}
\text{Bill and Sue: } & \quad 5 \times 2! = 10 \\
\text{Friends: } & \quad 4!
\end{align*}
\]

\[
10 \times 4! = 240
\]

(b) Sue must not sit next to Bill?

\[
\begin{align*}
\text{Total ways for everyone to sit down: } & \quad 6! \\
\text{# of ways Bill and Sue sit together: } & \quad 240
\end{align*}
\]

\[
6! - 240 = 480
\]

(c) Sue sits on one end of the row and Bill sits on the other end of the row?

\[
\frac{2}{5!} \quad \frac{4!}{5!} \quad \frac{1}{5!} = 2 \times 4! = 48
\]
4. Susie has 30 different songs (15 rock, 5 rap, 6 country, and 4 classical) she wants to arrange on her MP3 player.

(a) How many different playlists of all her songs are possible?

\[ 30! \approx 2.6525 \times 10^{32} \]

(b) How many different playlists are possible if all songs of the same genre must be grouped together?

\[ 4! \times (15! \times 5! \times 6! \times 4!) \approx 6.5078 \times 10^{19} \]
5. A jewelry store chain with 8 stores in Georgia, 12 in Florida, and 10 in Alabama is planning to close 10 of these stores.

(a) In how many different ways can this be done?

\[ C\left(30, 10\right) = 30,045,015 \]

(b) The company decided to close 2 stores in Georgia, 5 in Florida, and 3 in Alabama. In how many different ways can this be done?

\[ \frac{C\left(8, 2\right) \times C\left(12, 5\right) \times C\left(10, 3\right)}{GA \times FL \times AL} = 2,661,120 \]
6. How many 4-person committees are possible from a group of 9 people if 

(a) There are no restrictions?

\[ \binom{9}{4} = 126 \]

(b) Both Jim and Mary must be on the committee?

\[ \frac{9 \text{ total}}{J+M: 2} \quad \frac{\binom{2}{2} \times \binom{7}{2}}{J/M \text{ others}} = \binom{21}{21} \]

(c) Either Jim or Mary (but not both) must be on the committee?

\[ \binom{2}{1} \times \binom{7}{3} = 70 \]

\[ \text{or} \quad \frac{1}{\text{Jim}} \times \frac{\binom{7}{3}}{\text{others (not Mary)}} + \frac{1}{\text{Mary}} \times \frac{\binom{7}{3}}{\text{others (not Jim)}} \]
7. You have a box that contains 3 red, 4 black, 2 green, and 5 purple balls. If you take a sample of three balls from the box, in how many ways can you get

(a) Exactly 2 black balls and one green ball?

\[ \frac{\binom{4}{2} \times \binom{2}{1}}{\binom{11}{3}} = 12 \]

(b) No purple balls?

\[ \binom{9}{3} = 84 \]

(c) At least two purple balls?

\[ \left[ \binom{5}{2} \times \binom{9}{1} \right] + \left[ \binom{5}{3} \right] = 100 \]

(d) Exactly 2 red balls or exactly one purple ball?

\[ \left[ \binom{3}{2} \times \binom{11}{1} \right] + \left[ \binom{5}{1} \times \binom{9}{2} \right] - \left[ \binom{2}{2} \times \binom{5}{1} \right] = 198 \]
8. How many different 20-digit numbers can be formed from two 5's, three 2's, five 1's, six 4's, two 3's, an 8 and a 9?

One option: 5522211114444443389

\[
\frac{20!}{(2! \cdot 3! \cdot 5! \cdot 6! \cdot 2!)} \approx 1.1733 \times 10^{12}
\]
9. An author is writing a new book. She has a list of 20 different female names and 15 different male names, from which she will choose her 10 character names. In how many different ways can she

(a) Choose her 10 characters?

\[ C(35,10) = 183,579,396 \]

(b) Choose half female and half male characters?

\[
\frac{SF}{SM} \quad C(20,5) \times C(15,5) = 46,558,512
\]

(c) Choose exactly 6 female characters and determine the order of appearance of all the characters in her book?

\[
\frac{6F}{4M} \quad C(20,6) \times C(15,4) \times 10! \approx 1.9199 \times 10^{14}
\]
10. From a group of 15 chefs, one executive chef, one pastry chef and 4 sous chefs must be chosen. Assuming all 15 chefs are qualified for all the positions, in how many different ways can the 6 chefs be chosen?

\[
\frac{15}{E} \times \frac{14}{P} \times \frac{C(13,4)}{S}
\]

\[
= \mathcal{P}(15,2) \times C(13,4)
\]

\[
= 150,150
\]
11. An exam consists of ten multiple choice questions (each with 5 possible choices), of which eight must be answered correctly to pass the exam. In how many different ways can a person who answers all the questions on the exam pass the exam?

To pass: 8 correct, 2 wrong OR 9 correct, 1 wrong OR 10 correct

\[
\left[ \binom{10}{8} \times \frac{1^8}{5^8} \times \binom{2}{2} \times \frac{4^2}{5^2} \right] + \left[ \binom{10}{9} \times 1^9 \times 1 \times \binom{1}{1} \times 4 \right] + \left[ \binom{10}{10} \times 1^0 \right] = 761
\]
12. An experiment consists of [tossing a fair 4-sided die] and [flipping a fair coin].

(a) Describe an appropriate sample space for this experiment.

\[ S = \{ H, \overline{H}, 2H, \overline{2H}, 3H, \overline{3H}, 4H, \overline{4H} \} \]

(b) Write the event, \( E \), that heads is tossed on the coin.

\[ E = \{ H, \overline{H}, 3H, \overline{4H} \} \]

(c) Write the event, \( F \), that tails is tossed on the coin or a 3 is rolled on the die.

\[ F = \{ T, \overline{T}, 3H, 3T, 4T \} \]

(d) Are \( E \) and \( F \) mutually exclusive events? Why or why not?

No, \( E \cap F = \{ 3H \} \neq \emptyset \)
13. A pair of fair 6-sided dice are rolled and the outcomes on each die are recorded.

(a) Describe an appropriate sample space for this experiment.

\[
S = \begin{cases} 
 11, 21, 31, 41, 51, 61, \\
 12, 22, 32, 42, 52, 62, \\
 13, 23, 33, 43, 53, 63, \\
 14, 24, 34, 44, 54, 64, \\
 15, 25, 35, 45, 55, 65, \\
 16, 26, 36, 46, 56, 66 
\end{cases}
\]

(b) Is this a uniform sample space? Why or why not?

Yes, since dice are fair \(\square\) has a \(\frac{1}{36}\) chance of occurring.

(c) How many simple events are associated with this experiment?

36 \(\{11, 12, 13, \ldots, 66\}\)

(d) How many total events are associated with this experiment?

\(2^{36} \approx 6.879 \times 10^{10}\)

(e) Write the event \(E\), that a sum of 7 is rolled.

\(E = \{16, 25, 34, 43, 52, 61\}\)

(f) Find \(P(E)\).

\(\frac{6}{36}\)

(g) Find the probability that a product of 8 is rolled or a 6 is rolled on one of the dice.

\[P(\text{prod=8} \cup 6) = P(\text{prod=8}) + P(6) = \frac{13}{36}\]
14. A chick is randomly chosen from a new box of multi-colored Peeps containing 3 yellow, 4 purple, 2 green, 1 red, and 6 pink chicks and its color is noted.

(a) Write a non-uniform probability distribution for this experiment.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Yellow</th>
<th>Purple</th>
<th>Green</th>
<th>Red</th>
<th>Pink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Prob.</td>
<td>(\frac{3}{16})</td>
<td>(\frac{4}{16})</td>
<td>(\frac{2}{16})</td>
<td>(\frac{1}{16})</td>
<td>(\frac{6}{16})</td>
</tr>
</tbody>
</table>

\(\text{Sum} = 16\)

(b) What is the probability that a green or red chick is chosen?

\[ P(\text{Green U Red}) = \frac{3}{16} \]

(c) What is the probability that a traditional yellow chick is not chosen?

\[ P(\text{Yellow}^c) = 1 - P(\text{Yellow}) = 1 - \frac{3}{16} = \frac{13}{16} \]
15. Suppose $P(E) = 0.4$, $P(F) = 0.5$, and $P(E \cup F) = 0.6$. Calculate the following.

(a) $P(E \cap F) = b \rightarrow 0.3$

$p(E \cup F) = p(E) + p(F) - p(E \cap F)$

$0.6 = 0.4 + 0.5 - p(E \cap F)$

$0.6 = 0.9 - p(E \cap F)$

$p(E \cap F) = 0.9 - 0.6 = 0.3$

(b) $P(E^C) = 1 - P(F)$

$= 1 - 0.5$

$= 0.5$

$P(E^C \cup F^C) = P[(E \cap F)^c]$

$= 1 - p(E \cap F)$

$= 1 - 0.3$

$= 0.7$

(d) $P(E^C \cap F) = p(F) - p(E \cap F)$

$= 0.5 - 0.3$

$= 0.2$

\[\begin{array}{c|c|c|c|c|c}
& E & F & E \cap F & E^C \cap F & E^C \cup F^C \\
\hline
a & 0.4 & 0.5 & 0.3 & 0.1 & 0.7 \\
b & 0.1 & 0.3 & 0.2 & 0.1 & 0.5 \\
c & 0.2 & 0.2 & 0.1 & 0.1 & 0.7 \\
d & 0.1 & 0.2 & 0.1 & 0.1 & 0.7 \\
\hline
S & 0.4 & 0.2 & 0.5 & 0.3 & 1.0 \\
\end{array}\]
16. 200 students were surveyed about their regular vending machine purchases. 150 regularly purchase drinks, 75 regularly purchase snacks, and 30 don't make regular vending machine purchases. What is the probability that on a regular basis a randomly selected surveyed student

(a) Purchases both drinks and snacks?

\[
b \rightarrow \frac{55}{200}
\]

(b) Purchases only drinks?

\[
a \rightarrow \frac{95}{200}
\]

(c) Does not purchase drinks?

\[
c + d \rightarrow \frac{50}{200}
\]

\[
\begin{align*}
a + (b + c) + d & = 200 \rightarrow a = 95 \\
95 + b & = 150 \rightarrow b = 55 \\
55 + c & = 75 \rightarrow c = 20 \\
d & = 30
\end{align*}
\]
17. The students in a statistics class were surveyed concerning their eye color. The findings were gathered in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Blue Eyes $(B)$</th>
<th>Green Eyes $(G)$</th>
<th>Brown Eyes $(W)$</th>
<th>Other $(O)$</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females $(F)$</td>
<td>15</td>
<td>8</td>
<td>25</td>
<td>3</td>
<td>51</td>
</tr>
<tr>
<td>Males $(M)$</td>
<td>0</td>
<td>5</td>
<td>12</td>
<td>7</td>
<td>34</td>
</tr>
<tr>
<td>Totals</td>
<td>25</td>
<td>13</td>
<td>37</td>
<td>10</td>
<td>85</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected surveyed statistics student

(a) Is female? \[ P(F) = \frac{51}{85} \]

(b) Has blue eyes? \[ P(B) = \frac{25}{85} \]

(c) Is female and has blue eyes? \[ P(F \cap B) = \frac{15}{85} \]

(d) Is female or has blue eyes? \[ P(F \cup B) = P(F) + P(B) - P(F \cap B) = \frac{51 + 25 - 15}{85} = \frac{61}{85} \]

(e) Does not have green eyes? \[ P(G^c) = 1 - P(G) = 1 - \frac{13}{85} = \frac{72}{85} \]

(f) Is a male or does not have brown eyes? \[ P(M \cup W^c) = \frac{60}{85} \]
18. The following system of inequalities are constraints in a linear programming problem. Graph the feasible region. Label all lines and corner points.

\[
\begin{align*}
    x - y & \geq 0 \\
    x + y & \leq 10 \\
    10x + 9y & \geq 45 \\
    x & \geq 0 \\
    y & \geq 0
\end{align*}
\]

**Pick another x-value, say x = 1:**

\[
1 - y \geq 0 \implies y = 0 \implies (1, 1)
\]

(10, 0): 10 - 0 \geq 0 \text{ true}

\[
\begin{align*}
    x + y & \leq 10 \\
    x & \geq 0 \\
    y & \geq 0
\end{align*}
\]

(0, 0): 0 + 0 \leq 10 \text{ TRUE}

(10, 0): 10 + 0 \geq 45 \text{ FALSE}

**Corners:**

- **A:** (4.5, 0)
- **B:** \( x - y = 0 \)
- **C:** \( x - y = 0 \) \( x + y = 10 \) \( \left[ \begin{array}{l} 1 \ -1 \ 0 \\ 10 \ 9 \ 45 \end{array} \right] \left[ \begin{array}{l} 1 \ 0 \\ 0 \ 1 \end{array} \right] = \left[ \begin{array}{l} 45 \ 45 \\ 10 \ 10 \end{array} \right] \begin{pmatrix} 45/19 & 45/19 \end{pmatrix} \)
- **D:** (10, 0)
(a) At what point is the objective function \( f = 5x + 4.5y \) maximized on this region and what is the maximum value? (If not possible, explain why not.)

(b) At what point is the objective function \( f = 5x + 4.5y \) minimized on this region and what is the minimum value? (If not possible, explain why not.)

If \( S \) is bounded, you will always have a max + always have a min.

<table>
<thead>
<tr>
<th>Corners</th>
<th>( f = 5x + 4.5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: ((4.5, 0))</td>
<td>(5(4.5) + 4.5(0) = 22.5 \star)</td>
</tr>
<tr>
<td>B: ((\frac{45}{19}, \frac{45}{19}))</td>
<td>(22.5 \star)</td>
</tr>
<tr>
<td>C: ((5, 5))</td>
<td>(47.5)</td>
</tr>
<tr>
<td>D: ((10, 0))</td>
<td>(50 \star)</td>
</tr>
</tbody>
</table>

\(\star\) Max \( f = 50 \) and it occurs when @ \((10, 0)\)

\(\star\) Min \( f = 22.5 \) and it occurs at every pt on \( AB \)

\[
\begin{align*}
5x + 4.5y &= 22.5 \\
\frac{45}{19} &\leq x \leq 4.5
\end{align*}
\]
19. SET UP the following Linear Programming problem. DO NOT SOLVE.

You manage an ice cream factory that makes three flavors: Vanilla, Chocolate, and Strawberry. Into each batch of Vanilla go two eggs, one cup of milk and two cups of cream. Into each batch of Chocolate go one egg, one cup of milk and two cups of cream, while into each batch of Strawberry go one egg, two cups of milk and two cups of cream. You have in stock 220 eggs, 120 cups of milk, and 200 cups of cream. You make a profit of $3 on each batch of Vanilla, $2 on each batch of Chocolate, and $4 on each batch of Strawberry. How many batches of each flavor should you make in order to maximize your profit?

\[ \text{OBJ: } \text{Max } P = 3V + 2C + 4S \]

\[ \text{SUBJ TO: } \]
\[ \frac{2V + C + S}{\text{eggs used}} \leq 220 \] (eggs)
\[ \frac{V + C + 2S}{\text{c. of milk used}} \leq 120 \] (c. of milk)
\[ \frac{2V + 2C + 2S}{\text{c. of cream used}} \leq 200 \] (c. of cream)

\[ V \geq 0, C \geq 0, S \geq 0 \]
20. A company is selling two perfumes, A and B, for $20 and $17 per ounce, respectively. It takes the company 3 hours and $12 to produce each ounce of perfume A and 1 hour and $15 for each ounce of perfume B. If the company has a total of 90 hours and $600 for production, and the company is not allowed to make more than 20 ounces of perfume A, how many ounces of each perfume should the company produce in order to maximize its revenue? Is anything leftover at the optimal production level?

\[ x = \text{# of oz. of Perfume A} \]
\[ y = \text{"} B \]
\[ R = \text{revenue \ (in \$)} \]

**OBJ:** Max \( R = 20x + 17y \)

**SUBJ TO:**
\[ 3x + y \leq 90 \ (\text{hrs}) \]
\[ 12x + 15y \leq 600 \ ($\) \]
\[ 0 \leq x \leq 20 \quad y \geq 0 \]

Corners

A: (0,0) \hspace{1cm} R = 0
B: (0,40) \hspace{1cm} 680
C: (20,24) \hspace{1cm} 808
D: (20,0) \hspace{1cm} 400

**SOLN:** Sell 20 oz of A + 24 oz of B to max rev. at $808

**LEFTOVERS?**

Hrs: \( 3 \times 20 + 1 \times 24 = 84 - 48 \hspace{1cm} \text{hrs left} \)

\# Use: \( 12 \times 20 + 15 \times 24 = 600 - 180 \hspace{1cm} \text{left} \)
21. True or False. \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) and \( A = \{0, 1, 2, 3, 4, 5\} \).

\[
\begin{array}{cccc}
\text{T} & \text{F} & \emptyset \in A & \text{T} & \text{F} & n(A) = 5 & \text{T} & \text{F} & n(\{3, 4\}) = 2 \\
\text{T} & \text{F} & \emptyset \subseteq A & \text{T} & \text{F} & \{1, 3, 5\} \in A & \text{T} & \text{F} & n(\emptyset) = 1 \\
\text{T} & \text{F} & \{1, 2, 3\} \subset A & \text{T} & \text{F} & 2 \in A & \text{T} & \text{F} & 10 \in A^c \\
\text{T} & \text{F} & 2 \subseteq A & \text{T} & \text{F} & \{\emptyset\} = \emptyset & \text{T} & \text{F} & 0 = \emptyset \\
\end{array}
\]

\[ n(\emptyset) = 0 \]

\( \emptyset = \) empty set = \( \exists \exists \) \( \emptyset \) = \emptyset

\( \in = \) is an element of

(\( \subseteq \) is a subset)

(\( \subseteq \subseteq \) is a proper subset)

\( \notin = \) is NOT an element of

\( \subseteq = \) subset

\( \subseteq = \) set

\( \subseteq = \) subset

\( \subseteq = \) set

1 \( \subseteq A \)

\( \exists \subseteq \subseteq 1, 3, 5 \subseteq A \)

3 \( \subseteq A \)

\( \exists \subseteq \subseteq 1, 3, 5 \subseteq A \)

5 \( \subseteq A \)

\( \exists \subseteq \subseteq 1, 3, 5 \subseteq A \)

\( A^c \subseteq \text{ complement in } U \), but not in A
22. \( A = \{k, l, b\} \)

(a) How many total subsets does the set \( A \) have?

\[ 2^3 = 8 \]

(b) List all the subsets of \( A \).

\( \emptyset, \{k\}, \{l\}, \{b\}, \{k, l\}, \{k, b\}, \{l, b\}, \{k, l, b\} \)

(c) How many of the subsets are proper subsets? \((2^n) - 1\)

\[ 7 = (2^3) - 1 \]

(d) Give an example of two subsets of \( A \) that are disjoint. If this is not possible, then explain why not.

\( \{k, l\}, \{b\} \)  
\( A \cap B = \emptyset \)  
\( n(A \cap B) = 0 \)
23. Shade the part of a Venn diagram that is represented by \((A^C \cup B^c) \cap (C \cup A)\)

\[
A^c = \{c, f, g, h\} \\
B = \{b, c, e, f\} \\
A^c \cup B = \{b, c, e, f, g, h\} \\
A^c \cup B^c \cap (C \cup A) = \Omega
\]

\[
C = \{d, e, f, g\} \\
A = \{a, b, d, e\} \\
A \cup C = \{a, b, d, e, f, g\} \\
A \cap C = \{b, e, f, g\}
\]

24. Write the set notation that would represent the shaded portion of the Venn diagram.

\[
a = A \cap B^c \cap C^c \\
c = B \cap A^c \cap C^c
\]

\[
(A \cap B^c \cap C^c) \cup (B \cap A^c \cap C^c)
\]
25. $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{3, 5, 7\}$, $B = \{2, 4, 7\}$, and $C = \{2, 4, 6, 8\}$. Compute the following.

(a) $(A \cap B) \cup C$

\[
A \cap B = \{1, 7\}
\]

\[
U \cap C = \{2, 4, 6, 8\}
\]

\[
\Rightarrow \{1, 2, 4, 6, 7, 8\}
\]

(b) $A^c \cap B$

\[
A^c = \{0, 2, 4, 6, 8\}
\]

\[
B = \{1, 2, 4, 7, 8\}
\]

\[
\Rightarrow \{2, 4, 8\}
\]

(c) $A \cap (B \cup C)^c$

\[
(B \cup C)^c = \{1, 2, 4, 7, 8, 6\}
\]

\[
A = \{1, 3, 5, 7, 9\}
\]

\[
\Rightarrow \{3, 5, 9\}
\]
26. In a survey of 300 high school seniors:
   120 had not read Macbeth but had read As You Like It or Romeo and Juliet.
   61 had read As You Like It but not Romeo and Juliet.
   15 had read Macbeth and As You Like It.
   14 had read As You Like It and Romeo and Juliet.
   9 had read Macbeth and Romeo and Juliet.
   5 had read Macbeth and Romeo and Juliet but not As You Like It.
   40 had read only Macbeth.

Let \(M = \text{Macbeth}, R = \text{Romeo and Juliet}, \) and \(A = \text{As You Like It}.

(a) Fill in a Venn diagram illustrating the above information.

(b) How many students read exactly one of these books?
\[
a + c + g = 150
\]

(c) How many students did not read Romeo and Juliet?
\[
a + d + g + h = 221
\]

(d) How many students read Romeo and Juliet and also Macbeth or As You Like It?
\[
b + e + f = 19
\]

(e) Compute the value of \(n(M \cup (R^C \cap A))\) and write a sentence describing this information in words.
\[
\begin{align*}
\land a = \{a, b, c, d, e, f\} \land A = \{a, b, c, d, e, f\} \\
R^C = \{d, g, h\} \lor \{a, b, d, e, f\} \implies n\{a, b, d, e, g, h\} = 110
\end{align*}
\]

110 surveyed students read Macbeth OR read As You Like It but not R+J

(f) Compute the value of \(n(A^C \cap (R \cup M))\) and write a sentence describing this information in words.
\[
\begin{align*}
R \cup M = \{a, b, c, d, e, f\} \land A = \{a, b, c, d, e, f\} \\
R^C = \{d, g, h\} \land \land a = \{a, b, c, d, e, f\} = 1 \implies n\{a, b, c\} = 105
\end{align*}
\]

105 read R+J or Macbeth but do not read As You Like It