Review Problems for Final Exam

Note: This review does not cover every concept that could be tested on a final exam. Please also take a look at the previous Week-in-Reviews for more practice problems.

Chapter 9

1. Determine whether the following matrices are regular.

(a) \[
\begin{bmatrix}
0.7 & 0.2 \\
0.2 & 0.6 \\
0.1 & 0.2
\end{bmatrix}
\]
not stochastic \(\Rightarrow\) not regular

(b) \[
\begin{bmatrix}
0.8 & 1 & 0.4 \\
0.1 & 0 & 0.3 \\
0.1 & 0 & 0.3
\end{bmatrix}
\]
\[\frac{2}{3} = \begin{bmatrix} 0.78 & 0.8 & 0.74 \\
0.11 & 0.1 & 0.13 \\
0.11 & 0.1 & 0.13 \end{bmatrix}\]
stochastic + \(\Rightarrow\) regular

(c) \[
\begin{bmatrix}
1 & 0.2 & 0.5 \\
0 & 0.6 & 0.3 \\
0 & 0.2 & 0.2
\end{bmatrix}
\]
not stochastic \(\Rightarrow\) not regular

- Regular
  - Stochastic:
    - Every col sums to 1
    - All entries are non-negative
  - Power of the matrix has all positive entries

All powers contain 1 in 1st column
Chapter 9

2. Suppose that a study of diet soda drinkers found that currently 75% of people drink Diet Dr Pepper and 25% drink Diet Coke. Every year, 72% of those who drink Diet Dr Pepper will continue to do so, while the rest will switch to Diet Coke. Further, 54% of those who drink Diet Coke will continue to do so, while the rest will switch to Diet Dr Pepper.

(a) What percentage of soda drinkers will Diet Dr Pepper and Diet Coke have after 3 years?

\[
T = \begin{bmatrix}
  P & C \\
  0.72 & 0.46 \\
  0.28 & 0.54
\end{bmatrix}
\]

\[
X_0 = \begin{bmatrix}
  0.75 \\
  0.25
\end{bmatrix}
\]

\[
X_3 = T^3 X_0 = \begin{bmatrix}
  0.623878 \\
  0.376122
\end{bmatrix}
\]

\[
\Rightarrow 62.3878\% \text{ Diet DP} \quad 37.6122\% \text{ Diet Coke}
\]
(b) In the long run, what fraction of diet soda drinkers will Diet Dr Pepper and Diet Coke each have?

\[
X = \begin{bmatrix}
\text{p} \\
\text{c}
\end{bmatrix}, \quad \mathbf{T}X = X
\]

\[
\begin{pmatrix}
0.72 & 0.46 \\
0.28 & 0.54
\end{pmatrix}
\begin{pmatrix}
\text{p} \\
\text{c}
\end{pmatrix}
= \begin{pmatrix}
\text{p} \\
\text{c}
\end{pmatrix},
\]

\[
\begin{pmatrix}
0.72p + 0.46c \\
0.28p + 0.54c
\end{pmatrix}
\]

\[
0.72p + 0.46c = p \\
0.28p + 0.54c = c \\
p + c = 1
\]

\[
\begin{pmatrix}
-0.28 & 0.46 \\
0.28 & -0.46 \\
1 & 1
\end{pmatrix}
\begin{pmatrix}
p \\
c
\end{pmatrix}
= \begin{pmatrix}
23/37 \\
14/37 \\
0
\end{pmatrix}
\]

\[
p = \frac{23}{37} \text{ (Diet Dr Pepper)}
\]

\[
c = \frac{14}{37} \text{ (Diet Coke)}
\]
3. A company making radios finds that the total cost of producing 100 radios is $9,000 and that the total cost of producing 150 radios is $13,000. Each radio sells for $110. Let \( x \) be the number of radios made and sold. Find the

(a) cost function.
\[
C - 9000 = 80(x - 100)
\]
\[
C(x) = 80x + 1000
\]

(b) revenue function.
\[
R(x) = 110x
\]

(c) profit function.
\[
P = R - C
\]
\[
P(x) = 30x - 1000
\]

(d) break-even point and explain its meaning.
\[
R = C
\]
\[
110x = 80x + 1000
\]
\[
30x = 1000
\]
\[
x = \frac{1000}{30} = \frac{100}{3}
\]

\[
R\left(\frac{100}{3}\right) = 110\left(\frac{100}{3}\right) = \frac{11000}{3}
\]

\[
B.E. Pt: \left(\frac{100}{3}, \frac{11000}{3}\right)
\]

\[
\approx (33.33, 3666.67)
\]

\[
B.E. quant: \frac{100}{3} \text{ radios (in N)}
\]

\[
\Rightarrow \text{NO true b.E pt (can't sell part of a radio)}
\]
4. Use the Method of Corners to solve the following:

**OBJ:** Max $P = 2x + 5y$

**SUBJ TO:** $x + y \leq 10$

\[
\begin{align*}
3x + y & \geq 12 \\
-2x + 3y & \geq 3 \\
x & \geq 0, y \geq 0
\end{align*}
\]

\[x+y=10\]
\[\begin{array}{c}
x: (10, 0) \\
y: (0, 10) \\
(10,0): 0+0 \leq 10
\end{array}\]

\[3x+y\geq12\]
\[\begin{array}{c}
x: (4, 0) \\
y: (0, 12) \\
(0,10): 3(0)+0 \geq 12
\end{array}\]

\[-2x+3y\geq3\]
\[\begin{array}{c}
x: (-\frac{3}{2}, 0) \\
y: (0, \frac{1}{3}) \\
(0,0): -2(0)+3(0) \geq 3
\end{array}\]

\[
\begin{array}{c|c|c|c|c}
\text{CP} & P = 2x + 5y \\
\hline
(3, 3) & 2(3)+5(3) = 21 \\
(1, 9) & 47 \\
(5.4, 16) & 32.8 \\
\end{array}
\]
5. Given $U = \{0, 1, 2, \ldots, 10\}, A = \{1, 3, 5, 7, 9\}, B = \{2, 3, 4, 5, 6\}$, and $C = \{4, 8, 10\}$, find the following sets.

(a) $A \cup B^C = \{1, 3, 5, 7, 9\} \cup \{0, 1, 7, 8, 9, 10\} = \{0, 1, 3, 5, 7, 9, 10\}$

(b) $(C^C \cap A)^C$

(c) $B \cap (A^C \cup C)$
6. How many distinct ways can the letters in the word HULLABALOO be arranged?

\[
\text{Permutation of non-distinct objects}
\]

\[
\begin{align*}
\text{n = 10 total letters} \\
H & \ 1 \\
B & \ 1 \\
L & \ 3 \\
A & \ 2 \\
B & \ 1 \\
O & \ 2 \\
\end{align*}
\]

\[
\frac{10!}{(3!2!2!)} = 151,200
\]
7. A house costs $189,000. Bob makes a down payment of $12,000 and secures a loan for the remaining balance. The loan is to be amortized with monthly payments over 25 years at an annual interest rate of 6% compounded monthly.

(a) How much total interest will be paid on this loan?

\[
\text{Loan amount} = 189000 - 12000 = 177000
\]

\[
\text{Total Amount Paid:} \quad (1140.41)(300) = 342123
\]

\[
\text{Amount Borrowed:} \quad 177000
\]

\[
\text{Interest:} \quad 165123
\]

(b) Bob decides to refinance after 9 years. His new loan is a 15-year loan with an annual interest rate of 5% compounded monthly. What would be his new monthly payment?

\[
\text{New Loan}
\]

\[
\text{N} = 12 \times 15 = 180
\]

\[
\text{I} = 5
\]

\[
\text{PV} = 140541.88
\]

\[
\text{PMT} = ?
\]

\[
\text{FV} = 0
\]

\[
\text{P/Y} = \text{C/Y} = 12
\]

\[
\text{New Monthly Payment:} \quad 1111.40
\]
8. Suppose the weights of cats are normally distributed with an average weight of 8 pounds and a standard deviation of 1.75 pounds. What is the probability that a randomly selected cat weighs between 6 and 15 pounds?

\[ P(6 < X < 15) = \text{normalcdf}(6, 15, 8, 1.75) \approx 0.8734 \]
9. Solve the following for $a, b, c,$ and $d$.

\[4 \begin{bmatrix} -3 & 0 \\ 7 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ \frac{c-2}{b} \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} (d+4) \\ 5 \\ 0 \end{bmatrix}
\]

\[4 \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{7}{6} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ (c-2) \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} (d+4) \\ 5 \\ 0 \end{bmatrix}
\]

\[4 \begin{bmatrix} -\frac{2}{3} & \frac{4}{3} \\ \frac{19}{6} & -\frac{11}{3} \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 5 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ (c-2) \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} (d+4) \\ 5 \\ 0 \end{bmatrix}
\]

\[\begin{bmatrix} -\frac{8}{3} \\ \frac{16}{3} \end{bmatrix} + \begin{bmatrix} \frac{3}{3} \\ \frac{3b}{3c-6} \end{bmatrix} \begin{bmatrix} 3 \\ 12 \end{bmatrix} = \begin{bmatrix} (d+4) \\ 5 \\ 0 \end{bmatrix}
\]

\[\begin{bmatrix} \frac{1}{3} \\ \frac{3b+\frac{16}{3}}{3c+\frac{40}{3}} \end{bmatrix} = \begin{bmatrix} \frac{d+4}{5} \\ \frac{10}{a} \end{bmatrix}
\]

\[\frac{1}{3} = d+4
\]

\[-\frac{11}{3} = d
\]

\[3b + \frac{16}{3} = -2
\]

\[3b = -\frac{22}{3}
\]

\[b = \frac{-22}{9}
\]

\[3c + \frac{40}{3} = 5
\]

\[3c = \frac{-25}{3}
\]

\[c = \frac{-25}{9}
\]

\[-\frac{8}{3} = a
\]
10. There is a fruit market that has 120 oranges, 500 cherries, and 200 apples. Of these, there are 4 rotten oranges, 100 rotten cherries, and 10 rotten apples. What’s the probability that a customer will select 2 rotten oranges of 2 oranges he/she picked, 1 rotten apple of 1 apple he/she picked, and 30 rotten cherries of 40 cherries he/she picked?

\[
p(2 \text{ R or} \ 1 \text{ Rap} \ \& \ 30 \text{ Rch}) = p(2 \text{ R or}) \cdot p(1 \text{ Rap}) \cdot p(30 \text{ Rch}) \ \\
= \left[ \frac{C(4,2)}{C(120,2)} \right] \cdot \left[ \frac{C(10,1)}{C(200,1)} \right] \cdot \left[ \frac{C(100,30) \cdot C(400,10)}{C(500,40)} \right] \ \\
\approx 1.125 \times 10^{-39}
\]
11. It is known that 28% of a particular population enjoys eating seafood. From this population, 300 people are selected at random.

**BINOMIAL**  \( X = \# \text{ who enjoy seafood} \)  
\( n = 300 \)  
\( p = 0.28 \)  
\( q = 0.72 \)

(a) What is the probability that exactly 80 people enjoy eating seafood?

\[
p(X = 80) = \text{binompdf}(300, 0.28, 80) 
\approx 0.0455
\]

(b) What is the probability that at least 75 people, but no more than 125 people, enjoy eating seafood?

\[
P(75 \leq X \leq 125) = \text{binomcdf}(300, 0.28, 125) - \text{binomcdf}(300, 0.28, 74) 
\approx 0.8900
\]

(c) Use an appropriate normal distribution to approximate the probability that no more than 100 people enjoy eating seafood.

Since \( X \) is binomial,

\[
\mu = np = 300(0.28) = 84 \\
\sigma = \sqrt{npq} = \sqrt{300(0.28)(0.72)} = \sqrt{60.48}
\]

\[
P(0 \leq X \leq 100) \approx \text{normalcdf}(-0.5, 100.5, 84, \sqrt{60.48}) 
\approx 0.9831
\]
12. A simple economy consists of two sectors: food and shelter. The production of 1 unit of food requires the consumption of 0.4 units of food and 0.2 units of shelter. The production of 1 unit of shelter requires the consumption of 0.3 units of food and 0.2 units of shelter.

(a) Find the gross output of goods needed to satisfy a consumer demand of 12285 units of food and 3185 units of shelter.

\[ A = \begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.2 \end{bmatrix} \]

\[ D = \begin{bmatrix} 12285 \\ 3185 \end{bmatrix} \]

\[ X = \begin{bmatrix} F \\ S \end{bmatrix} \]

\[ X = A X + D \]

\[ X - A X = D \]

\[ (I - A)^{-1} (I - A) X = (I - A)^{-1} D \]

\[ X = (I - A)^{-1} D \]

\[ X = \begin{bmatrix} 25675 \\ 10400 \end{bmatrix} \]

\[ 25675 \text{ units of food} \]

\[ 10400 \text{ units of shelter} \]

(b) Find the internal consumption of goods while meeting the above demand.

\[ AX = X - D \]

\[ = \begin{bmatrix} 25675 \\ 10400 \end{bmatrix} - \begin{bmatrix} 12285 \\ 3185 \end{bmatrix} = \begin{bmatrix} 13390 \\ 7215 \end{bmatrix} \]

\[ 13390 \text{ units of food} \]

\[ 7215 \text{ units of shelter} \]
13. You pay $5.00 to play the following game. You have 2 chances to draw a ball from a bag. If you draw a white ball, you win nothing and if you draw a purple ball you win six dollars. In the bag there are 30 balls total - 20 are white and 10 are purple. After the first ball is drawn it is replaced before the next ball is drawn.

(a) Find the expected net winnings of this game.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Win</th>
<th>Net Win</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} w_1 )</td>
<td>0</td>
<td>-5</td>
<td>( \frac{2}{3} (\frac{2}{3}) = \frac{4}{9} )</td>
</tr>
<tr>
<td>( \frac{1}{3} p_1 )</td>
<td>6</td>
<td>1</td>
<td>( \frac{1}{3} (\frac{2}{3}) = \frac{2}{9} )</td>
</tr>
<tr>
<td>( \frac{2}{3} w_2 )</td>
<td>6</td>
<td>7</td>
<td>( \frac{2}{9} )</td>
</tr>
<tr>
<td>( \frac{1}{3} p_2 )</td>
<td>12</td>
<td>7</td>
<td>( \frac{1}{9} )</td>
</tr>
</tbody>
</table>

(b) Is this game fair? Why or why not?

No, for a game to be fair, expected net win must be zero.
14. You are given the following data: (0,0), (1,2), (3,5), (4,6), and (6,9), where $x$-values represent the number of items sold (in hundreds) and $y$-values represent the amount of profit made (in thousands of dollars).

(a) Find the least-squares line for the data.

$$ x \to L_1: 0, 1, 3, 4, 6 $$

$$ y \to L_2: 0, 2, 5, 6, 9 $$

LinReg $y_1$  \Rightarrow  $y_1 \approx 1.4699x + 0.2982$

(b) Is the line you found a good fit for the data? Why or why not?

$$ r \approx 0.9972 $$

If $|r| \geq 0.8$, then the line is considered a good fit.  \Rightarrow \text{Yes}

(c) Use your line to predict the amount of profit made when 550 items are sold.

$$ x = \frac{550}{100} = 5.5 $$

$$ y_1(5.5) = 8.355263158 \times 1000 $$

$$ \approx 8355.26 $$

(d) Use your line to predict the amount of items which must be sold to generate a profit of $32,500.

$$ y = \frac{32500}{1000} = 32.5 $$

$$ y = \text{linreg line} \Rightarrow \text{calc-intersect} \Rightarrow (21.982036, 32.5) $$

$$ \times 100 \Rightarrow 2198 \text{ items} $$
15. A child wants to build a block city. Each house requires 50 square blocks, 100 rectangular blocks, and 4 windows. Each store requires 50 square blocks, 125 rectangular blocks, and 8 windows. Each school requires 100 square blocks, 75 rectangular blocks, and 20 windows. If there are 5250 square blocks, 7375 rectangular blocks, and 880 windows, how many houses, stores, and schools can the child build if all of the materials are to be used?

\[ H = \# \text{ of houses} \]
\[ S = \# \text{ of stores} \]
\[ L = \# \text{ of schools} \]

\[
\begin{array}{c|c|c|c|c}
& H & S & L & \text{Total} \\
\hline
\text{Sq} & 50 & 50 & 100 & 5250 \\
\text{Rect} & 100 & 125 & 75 & 7375 \\
\text{Win} & 4 & 8 & 20 & 880 \\
\end{array}
\]

\[
\begin{align*}
\text{sq:} & \quad 50H + 50S + 100L = 5250 \\
\text{rect:} & \quad 100H + 125S + 75L = 7375 \\
\text{wind:} & \quad 4H + 8S + 20L = 880
\end{align*}
\]

\[
\begin{bmatrix}
50 & 50 & 100 & 5250 \\
100 & 125 & 75 & 7375 \\
4 & 8 & 20 & 880 \\
\end{bmatrix}
\xrightarrow{\text{rref}}
\begin{bmatrix}
1 & 0 & 0 & 20 \\
0 & 1 & 0 & 25 \\
0 & 0 & 1 & 30 \\
\end{bmatrix}
\]

\[
H = 20 \text{ houses} \\
S = 25 \text{ stores} \\
L = 30 \text{ schools}
\]
16. Shade \((A^c \cap B)^c \cap C^c\).
17. Kathryn has a collection of 18 different Tweety Birds, 8 plush toys, 6 plastic figurines, and 4 porcelain figurines. She wants to arrange all of these on one shelf.

(a) How many total arrangements exist?

\[ 18! \approx 6.4024 \times 10^{15} \]

(b) How many total arrangements are possible if all Tweety Birds made out of the same material are grouped together?

\[ 3! \cdot (8! \cdot 6! \cdot 4!) = 4,180,377,600 \]
18. Find the value of $a$ if $P(Z > a) = 0.6235$, where $Z$ is the standard normal random variable.

$\mu = 0$

$\sigma = 1$

Area to LEFT

$a = \text{invNorm}(1 - 0.6235, 0, 1)$

$a \approx 0.3147$
19. The quantity demanded for model Corvettes is 8000 if the price is $20. If the price goes up to $25, the quantity demanded goes down to 6000. A manufacturer will not market the models if the price drops below $10. For every $5 increase, he will produce 2000 more models.

(a) Find the demand function. 

\[ \begin{align*} 
& (8000, 20) \quad (6000, 25) \\
& m = \frac{25-20}{6000-8000} = -\frac{1}{400} \\
& p = \frac{-1}{400} (x-8000) \\
& p = \frac{-1}{400} x + 20 \\
& p = \frac{-1}{400} x + 40 \\
\end{align*} \]

(b) Find the supply function. 

\[ \begin{align*} 
& (0, 10) \quad (2000, 15) \\
& m = \frac{15-10}{2000-0} = \frac{1}{400} \\
& p = \frac{1}{400} x + 10 \\
\end{align*} \]

(c) Find the equilibrium point.

\[ S = D \]

\[ \frac{1}{400} x + 10 = \frac{-1}{400} x + 40 \]

\[ \frac{2}{400} x = 30 \]

\[ x = 30 \left( \frac{400}{2} \right) = 6000 \]

\[ p = \frac{1}{400} (6000) + 10 = 25 \]

EQ PT: (6000, 25)
20. Are the following matrices in reduced row-echelon form?

(a) \[
\begin{bmatrix}
1 & -2 & 0 & t \\
0 & 1 & 3 & u \\
0 & 0 & 1 & v \\
\end{bmatrix}
\]

\[\text{NO, violates (4)}\]

(b) \[
\begin{bmatrix}
0 & 0 & 1 & 0 & x \\
1 & 0 & 0 & 0 & w \\
0 & 0 & 0 & 0 & t \\
0 & 0 & 1 & 0 & r \\
\end{bmatrix}
\]

\[\text{NO, violates (1)}\]

RREF

1. All zero rows below non-zero rows
2. First non-zero entry in each row must be 1 (leading 1)
3. Leading 1's go to the right as you move down matrix
4. A column containing a leading 1 should have all other entries equal to zero.
21. In 2002, 100 Aggies were surveyed concerning where they preferred to go on Friday nights.

- 35 liked Harry's and Shadow Canyon
- 69 liked The Chicken
- 59 liked more than one of the three places
- 30 liked Harry's or Shadow Canyon, but not The Chicken
- 15 liked Harry's and The Chicken, but not Shadow Canyon
- 30 liked all three places
- 15 liked Harry's, but not The Chicken
- 1 liked to stay home and study for math!

(a) Fill in an appropriate Venn diagram with the given information.

(b) Express “the Aggies who liked only Shadow Canyon” with set notation (set names, unions, intersections, or complements).

\[ S \cap H^c \cap C^c \]
22. In a group of 200 people in Florida, 3/4 are Republicans and 1/4 are Democrats. In the 2000 presidential election, 95% of the Republicans voted for Bush, 4% voted for Gore, and the rest had unreadable ballots. On the other hand, 60% of the Democrats voted for Gore, 10% voted for Bush, and the rest had unreadable ballots. If an unreadable ballot is selected at random, what is the probability it was cast by

(a) a Democrat?

\[
\Pr(D|u) = \frac{\Pr(D \cap u)}{\Pr(u)} = \frac{\frac{3}{4} \times 0.3}{\frac{3}{4} \times 0.01 + \frac{1}{4} \times 0.3} = \frac{1}{11}
\]

(b) a Republican?

\[
\Pr(R|u) = \frac{\Pr(R \cap u)}{\Pr(u)} = \frac{\frac{3}{4} \times 0.01}{\frac{3}{4} \times 0.01 + \frac{1}{4} \times 0.3} = \frac{1}{11}
\]
23. If the odds are 5 to 8 against an event occurring, what is the probability of
(a) the event occurring?

\[ P(E) = \frac{n(E)}{n(S)} = \frac{8}{5+8} = \frac{8}{13} \]

(b) the event not occurring?

\[ P(E^c) = \frac{n(E^c)}{n(S)} = \frac{5}{13} \]
24. Given \( A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \) \( B = \begin{bmatrix} 5 & 1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix} \) and \( C = \begin{bmatrix} 6 & 1 & 3 & 4 \\ 7 & 5 & 1 & 0 \\ 2 & 1 & 8 & 1 \end{bmatrix} \) find:

(a) \( AB \)

\( (2 \times 3)(3 \times 2) \)

\[
\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5a + 2b + 3c \\ 5d + 2e + 3f \end{bmatrix}
\]

(b) \( BC \)

\( (3 \times 2)(3 \times 4) \)

\[
\begin{bmatrix} 6 & 1 & 3 & 4 \\ 7 & 5 & 1 & 0 \\ 2 & 1 & 8 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1a + 0 + 4c \ \\ 1d + 0 + 4f \end{bmatrix} = \begin{bmatrix} a + 0 + 4c \\ d + 0 + 4f \end{bmatrix}
\]

\( \text{NOT POSSIBLE} \)
25. A company makes 100 CDs with 10 of them being defective. You buy 8 from the factory directly.

(a) In how many ways can you get exactly 6 defective CDs?

\[ \binom{10}{6} \binom{90}{2} = 841,050 \]

(b) In how many ways can you get at least 1 defective CD?

At least 1 D ⇒ 1D n 2D n 3D or ... or 8D

⇒ Total - \[ \binom{100}{8} \]

⇒ \[ \binom{100}{8} - \binom{10}{0} \binom{90}{8} \]

\[ \approx 1.0857 \times 10^{11} \]
26. Using the given Venn diagram, find the following:

(a) \[ P(D) = \frac{n(D)}{n(s)} = \frac{60}{90} = \frac{2}{3} \]

(b) \[ P(B \mid D) = \frac{P(B \cap D)}{P(D)} = \frac{n(B \cap D)}{n(D)} = \frac{20}{60} = \frac{1}{3} \]

(c) \[ P(D^c) = \frac{n(D^c)}{n(s)} = \frac{30}{90} = \frac{1}{3} \]

(d) \[ P(B \mid D^c) = \frac{n(B \cap D^c)}{n(D^c)} = \frac{10}{30} = \frac{1}{3} \]

(e) \[ P(B \cup D) = \frac{n(B \cup D)}{n(s)} = \frac{70}{90} = \frac{7}{9} \]
27. Solve the following systems of equations.

(a) \[
\begin{align*}
3x - 5y &= 6 \\
-2x + 4y &= -7 \\
2x - 4y &= 6
\end{align*}
\]

\[
\begin{bmatrix}
3 & -5 & | & 6 \\
-2 & 4 & | & -7 \\
2 & -4 & | & 6
\end{bmatrix}
\xrightarrow{\text{Ry}}
\begin{bmatrix}
x & y & | & 0 \\
0 & 1 & | & 0 \\
0 & 0 & | & 1
\end{bmatrix}
\]

\[x = 0, \quad y = 0, \quad 0 \neq 1 \quad \Rightarrow \text{NO SOLN}\]

(b) \[
\begin{align*}
2x - y - 3z &= 3 \\
2x + 2y - z &= 7 \\
3y + 2z &= 4
\end{align*}
\]

\[
\begin{bmatrix}
2 & -1 & -3 & | & 3 \\
2 & 2 & -1 & | & 7 \\
0 & 3 & 2 & | & 4
\end{bmatrix}
\xrightarrow{\text{Ry}}
\begin{bmatrix}
x & y & z & | & 13/6 \\
0 & 1 & 2/3 & | & 4/3 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

\[x - \frac{7}{6}z = \frac{13}{6} \quad \Rightarrow \quad x = \frac{13}{6} + \frac{7}{6}z\]

\[y + \frac{2}{3}z = \frac{4}{3} \quad \Rightarrow \quad y = \frac{4}{3} - \frac{2}{3}z\]

\[0 = 0 \quad \checkmark \]

SOLN:

\[(x, y, z) = \left( \frac{13}{6} + \frac{7}{6}t, \frac{4}{3} - \frac{2}{3}t, t \right) \]

\[t = \text{any real #}\]
28. SET UP the following linear programming problem, but do not solve.

A company produces two types of saddles, one english and one western. The english sells for $350 and the western for $600. The english saddle requires 5 units of leather, 12 hours assembly time, and 2 units of stitching. The western saddle requires 12 units of leather, 16 hours of assembly, and 4 units of stitching. The company only has 1100 units of leather, 32 eight-hour days for assembly, and 42 units of stitching. How many of each model should be produced in order for the company to maximize its revenue?

\[ E = \# \text{ of english saddles} \]
\[ W = \# \text{ of western saddles} \]
\[ R = \text{revenue (\$)} \]

\[
\text{OBJ: } \max R = 350E + 600W
\]

\[
\text{SUBJ TO: } \begin{align*}
5E + 12W & \leq 1100 \text{ (units of leather)} \\
12E + 16W & \leq 32 \times 8 \text{ (hrs of assembly)} \\
2E + 4W & \leq 42 \text{ (units stitching)} \\
E & \geq 0, \quad W & \geq 0
\end{align*}
\]
29. A poll is being conducted among readers of USA Today. Eight multiple choice questions are asked, each with 5 possible answers. In how many different ways can a reader complete the poll if exactly one response is given to each question?

\[
\underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 5^8 = 390,625
\]

30. All probabilities are between ___ and ___, inclusive.
31. A pair of fair 6-sided dice are rolled. If the sum of the numbers which lands uppermost on the dice is 6 or 7, what is the probability that the number which lands uppermost on the second die is a 4 or 5?

\[
P(\text{Second die is 4 or 5} \mid \text{Sum = 6 or 7}) = \frac{4}{11}
\]
32. The probability distribution of a random variable $X$ is given. Compute the mean, median, variance, standard deviation, and range of $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(X = x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

1-Variate Stats $L_1, L_2$

Mean = $\bar{X} = \begin{bmatrix} 2 \end{bmatrix}$

Median = $\text{Med} = \begin{bmatrix} 2 \end{bmatrix}$

Std Dev = $\sigma_X = \begin{bmatrix} 1 \end{bmatrix}$

Variance = $(\sigma_X)^2 = 1^2 = \begin{bmatrix} 1 \end{bmatrix}$

Range = $\max_x - \min_x + 1 = 4 - 1 + 1 = \begin{bmatrix} 4 \end{bmatrix}$