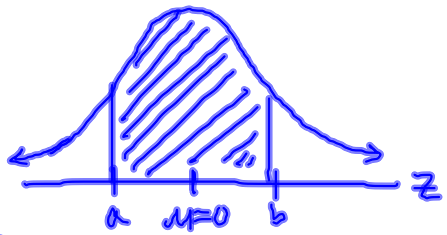
Week-In-Review #10 (8.5, 8.6, 5.1)

1. Let Z be the standard normal random variable. Calculate the following probabilities:

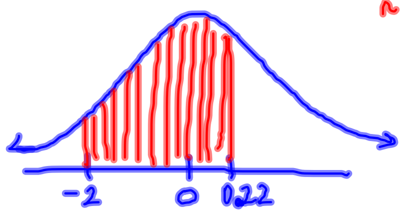
$\mu=0$ AND $\sigma=1$ (area)

normal R.V. are continuous R.V.

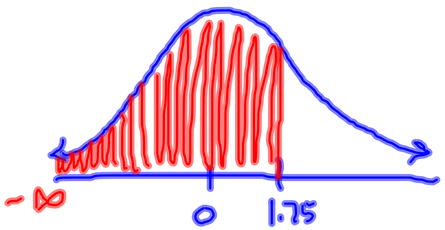


Area $= P(a \le Z \le b)$
$= P(a < Z < b)$
$= P(a \le Z < b)$
$= P(a < Z \le b)$
$=$ normalcdf(a, b, μ, σ)

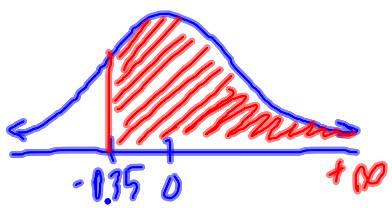
(a) $P(-2 \le Z < 0.22)$ = normalcdf(−2, 0.22, 0, 1) $\approx \boxed{0.5643}$



(b) $P(Z \le 1.75)$ = normalcdf(−1E99, 1.75, 0, 1) $\approx \boxed{0.9599}$



(c) $P(Z > -0.35)$ = normalcdf(−0.35, 1E99, 0, 1) $\approx \boxed{0.6368}$



2. Let Z be the standard normal random variable. Find a such that

$\mu = 0$ AND $\sigma = 1$

(a) $P(Z \le a) = 0.8158$

$a = \text{invNorm}(0.8158, 0, 1)$

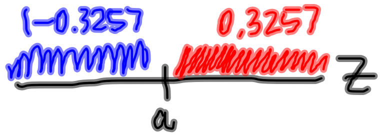
$\boxed{a \approx 0.8995}$

CK: $P(Z < 0.8995) = ?$

normalcdf$(-1E99, 0.8995, 0, 1)$

≈ 0.8158

(b) $P(Z \ge a) = 0.3257$

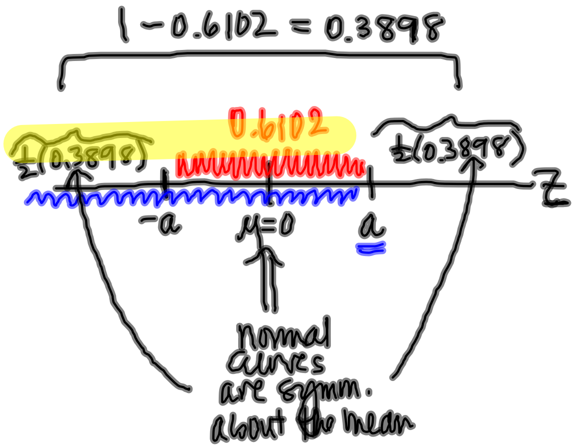


$a = \text{invNorm}(1 - 0.3257, 0, 1)$

$\boxed{a \approx 0.4518}$

invNorm(area, μ, σ) gives the value of a so that $P(X < a) = $ area

(c) $P(-a < Z < a) = 0.6102$



$a = \text{invNorm}\left(\frac{1}{2}(0.3898) + 0.6102, 0, 1\right)$

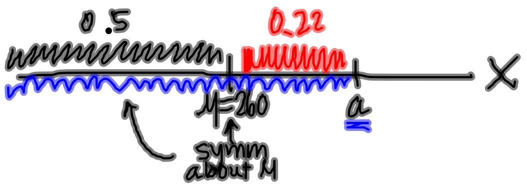
$\boxed{a \approx 0.8600}$

3. Let X be a normal random variable with $\mu = 260$ and $\sigma = 35$. Find each of the following.

(a) $P(X < 200)$ $=$ normalcdf$(-1E99, 200, 260, 35) \approx \boxed{0.0432}$

(b) $P(X \geq 180)$ $=$ normalcdf$(180, 1E99, 260, 35) \approx \boxed{0.9889}$

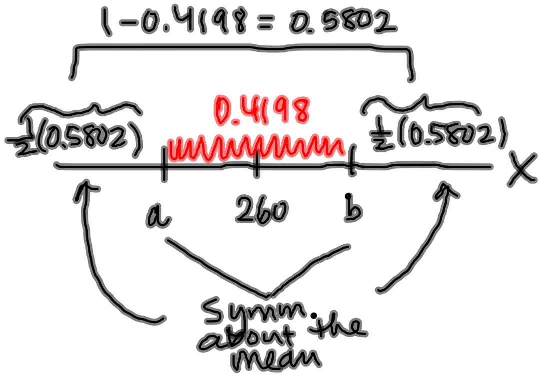
(c) The value of a such that $P(260 < X < a) = 0.22$



$a =$ invNorm$(0.5+0.22, 260, 35)$

$\boxed{a \approx 280.3994}$

(d) The values of a and b such that $P(a < X < b) = 0.4198$ if a and b are symmetric about the mean.



$a =$ invNorm$(\frac{1}{2}(0.5802), 260, 35)$

$\boxed{a \approx 240.6418}$

$b =$ invNorm$(\frac{1}{2}(0.5802)+0.4198, 260, 35)$

$\boxed{b \approx 279.3582}$

4. Suppose weights of bags of snack mix are normally distributed with a mean of 10 ounces and a standard deviation of 0.6 ounces. What is the probability that a bag selected at random weighs

X

$\mu = 10$ oz

$\sigma = 0.6$ oz

(a) Between 9.5 and 11 ounces?

$$P(9.5 < X < 11) = \text{normalcdf}(9.5, 11, 10, 0.6) \approx \boxed{0.7499}$$

(b) At least 9 ounces?

$$P(X \geq 9) = \text{normalcdf}(9, 1E99, 10, 0.6) \approx \boxed{0.9522}$$

(c) Less than 8.5 ounces?

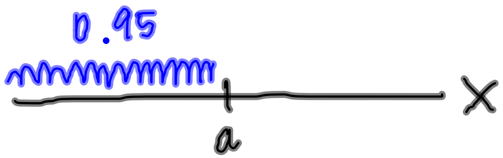
$$P(X < 8.5) = \text{normalcdf}(-1E99, 8.5, 10, 0.6) \approx \boxed{0.0062}$$

5. A study finds that the lifespan of phone batteries (X) are normally distributed with a mean of 2 years and a standard deviation of 1.5 months. $\sigma = 1.5$ mo. $\mu = 24$ mo.

(a) What is the probability that a phone battery will have a lifespan between 22 and 26 months?

$$P(22 < X < 26) = \text{normalcdf}(22, 26, 24, 1.5) \approx \boxed{0.8176}$$

(b) What battery lifespan ("a", x-value) corresponds to the $\underbrace{95^{th} \text{ percentile}}_{\text{95\% of batteries have lifespan less than this value.}}$?



$$a = \text{invNorm}(0.95, 24, 1.5)$$

$$\boxed{a \approx 26.4673 \text{ mo.}}$$

6. Find the amount of money at the end of 5 years on a \$200 deposit in an account paying simple interest at a rate of 4.75% per year. How much interest is earned?

$\Downarrow$

$I = Prt$

$A = P(1+rt)$

r as a decimal

A $A = P(1+rt)$

$= 200(1+0.0475(5))$

$= \boxed{\$247.50}$

B $I = Prt$

$= 200(0.0475)(5)$

$= \boxed{\$47.50}$

or....

$A = P + I$

$\Rightarrow I = A - P$

$= 247.50 - 200$

$= \boxed{\$47.50}$

7. A bank deposit paying simple interest grew from an initial sum of \$2000 to a sum of \$2150 in 7 months. Find the interest rate.

$7\text{ mo} \cdot \frac{1\text{ yr}}{12\text{ mo}}$

$\Rightarrow \frac{7}{12}\text{ yr}$

$A = P(1+rt)$

$\frac{2150}{2000} = \frac{\cancel{2000}\left(1+r\left(\frac{7}{12}\right)\right)}{\cancel{2000}}$

$1.075 = 1 + \frac{7}{12}r$

$\left(0.075 = \frac{7}{12}r\right)\frac{12}{7}$

$r = 0.075\left(\frac{12}{7}\right)$

$= 0.12857\ldots$

$\boxed{\text{Rate} = 12.857\%/\text{yr}}$

8. How much money will be in an account after 2 years on a \$500 deposit that earns interest at a rate of 5% per year compounded continuously?

$$A = Pe^{rt}$$
$$= 500e^{0.05(2)}$$
$$= \boxed{\$552.59}$$

How much interest earned?

$552.59 - 500$
$= \$52.59$

9. If Mark invests \$5000 into an account paying interest at a rate of 8% per year compounded monthly, how much money will he have at the end of 5 years (assuming no additional deposits or withdrawals)?

$m = 12$

TVMSolver
N = mt
I = int. rate
PV = P
PMT = pmt amt
FV = A
P/Y = C/Y = m
ALPHA ENTER to solve

N = 12(5) = 60
I = 8
PV = −5000 \$ leaving Mark's hands
PMT = 0
FV = ? → $\boxed{\$7449.23}$
P/Y = C/Y = 12

m = # of times compd / yr
annually ⇒ m = 1
semi-annually ⇒ m = 2
quarterly ⇒ m = 4
monthly ⇒ m = 12
weekly ⇒ m = 52
daily ⇒ m = 365

10. In 18 months Brian needs \$1750 in order to buy a specific computer. If he finds an account paying interest at a rate of 5.95% per year compounded weekly, how much could he invest now in order to have the money he needs for the computer?

$m = 52$

$\frac{18 \text{ mo}}{} \left| \frac{1 \text{ yr}}{12 \text{ mo}} \right. = 1.5 \text{ yr}$

TVMSolver

N = 52(1.5)
I = 5.95
PV = ? $\rightarrow$ $\boxed{\$1600.66}$
PMT = 0
FV = 1750
P/Y = C/Y = 52

11. Kevin inherits \$50,000. If he invests it by placing it into an account paying interest at a rate of 10.5% per year compounded monthly, how long would he have to leave his money in the account before having \$100,000?

$m = 12$

TVMSolver

N = ? $\longrightarrow$ N = 79.56..... $\approx$ $\boxed{80 \text{ mo}}$ $\Rightarrow$ $\frac{80}{12}$ yrs
I = 10.5
PV = −50000
PMT = 0
FV = 100000
P/Y = C/Y = 12

12. What interest rate, $\underbrace{\text{compounded daily}}_{m=365}$, will $\underbrace{\text{quadruple}}_{4x}$ \$1200 after 5 years?

TVMSolver

N = 365 (5)

I = ? $\rightarrow$ $\boxed{27.7364\%/\text{yr}}$

PV = −1200

PMT = 0

FV = 4(1200) = 4800

P/Y = C/Y = 365

13. A major credit card company has a finance charge of 1.5% $\overset{m=12}{\text{per month}}$ on the outstanding indebtedness. Susie charged \$1000 and did not pay her bill for $\underbrace{\text{6 months}}$. What is the bill after the 6 months?

$$\frac{1.5\%}{\text{mo}} \cdot \frac{12\text{ mo}}{1\text{ yr}} = 18\%\text{ yr}$$

$$\frac{6\text{ mo}}{} \cdot \frac{1\text{ yr}}{12\text{ mo}} = \frac{1}{2}\text{ yr}$$

TVMSolver

N = 12(½) = 6

I = 18

PV = 1000

PMT = 0

FV = ? $\rightarrow$ $\boxed{\$1093.44}$

P/Y = C/Y = 12

14. Which account would be a better account for an investment? For a credit card?

OPTION A: 9% per year, compounded monthly $m=12$

OPTION B: 8.8% per year, compounded daily $m=365$

OPTION C: 8.9% per year, compounded continuously

Use effective rates of interest to compare accts.

(A) $EFF(r,m) = EFF(9,12) \approx 9.3807\%$ → best investment

(B) $EFF(8.8,365) \approx 9.1977\%$ → best credit card rate

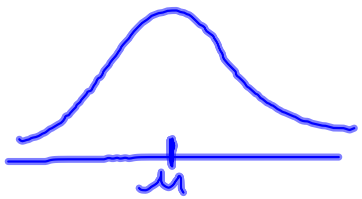
(C) $r_{eff} = \underbrace{(e^{r})-1}_{\times 100} = \underbrace{\left(e^{0.089}\right)-1}_{\times 100} \approx 9.3081\%$

Investment ⇒ want highest yield

Credit Card ⇒ want lowest yield

15. Determine whether the following statements are True or False.

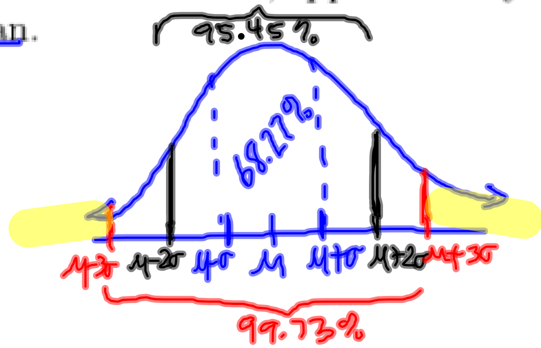
(a) A normal curve with a higher variance is taller than one with a lower variance.



highly spread out from the mean

FALSE → higher variance means more area under "tails" so less area near mean (shorter)

(b) In a normal distribution, approximately 99.73% of the data lies further than 3 standard deviations from the mean.



FALSE → 99.73% of the data lies WITHIN 3 std. dev. of the mean

(c) A normal random variable with a mean of zero is the standard normal curve.

$\mu=0$

FALSE → $\mu=0$ AND $\sigma=1$ for std. normal

(d) The more times a year an account is compounded, the more interest that is earned. (Assume int rate the same.)

TRUE