Week-In-Review #4 (3.1, 3.2, 3.3)

1. Graph to find the solution set for each of the following systems of inequalities. Find all corner points and state whether the solution set is bounded or unbounded.

(a) \[ x - y \leq 10 \]
\[ 2x + 5y \geq 20 \]
\[ x \geq 4 \]
\[ y \geq 1 \]

\[ \text{Graph} \]

\[ \text{x-inter: } (10, 0) \]
\[ \text{y-inter: } (0, -10) \]
\[ (0,0): \ 0 - 0 \leq 10 \]
\[ 0 \leq 10 \text{ true} \]

\[ \text{S} \]

\[ \text{Corners} \]
\[ A: x = 4 \]
\[ 2x + 5y = 20 \]
\[ 2(4) + 5y = 20 \]
\[ 5y = 12 \]
\[ y = \frac{12}{5} \]

\[ B: y = 1 \]
\[ 2x + 5y = 20 \]
\[ 2x + 5(1) = 20 \]
\[ 2x = 15 \Rightarrow x = \frac{15}{2} \]

\[ C: y = \frac{17}{2} \]
\[ x - y = 10 \]
\[ x - 1 = 10 \]
\[ x = 11 \]
(b) \(-0.5x + 4y \leq 5\)
\[-6 \leq x \leq 4\]
\[y \geq -8\]

\[y \leq -2x\]
\[x: \left( \frac{10}{2}, 0 \right)\]
\[y: \left( 0, \frac{5}{4} \right)\]
\[(0,0): 0 \leq 5\] True

Another pt on the line \(y = -2x\)
Say \(x = 1 \rightarrow y = -2(1)\)
\((-1, -2)\)
Test: \((10, 0) \rightarrow 0 \leq -2(10)\)
\[0 \leq -20\] False

\(-6 \leq x \leq 4\)
\[x \geq -6, x \leq 4\]

\[y \geq -8\]

Bounding:

Corners:
\(A\) \(x = -6, y = -8\) \((-6, -8)\)
\(B\) \(x = -6\)
\(-0.5x + 4y = 5\) \((-6, \frac{5}{4})\)
\(-0.5(-6) + 4y = 5\)
\[3 + 4y = 5\] \(4y = 2\) \(y = \frac{1}{2}\)

\(C\) \(y = -2x\)
\(-0.5x + 4y = 5\) \((-10/11, 2x)\)

\[y = -2x\]
2. Solve the following linear programming (LP) problem:

(a) OBJECTIVE: Maximize \( R = 15x + 9y \)

SUBJECT TO:
\[
\begin{align*}
  x + y &\leq 5 \\
  -x + 2y &\leq 6 \\
  3x - 4y &\leq 12 \\
  x &\geq 0, \quad y \geq 0
\end{align*}
\]

Graph these inequalities:

- \( x + y \leq 5 \)
- \( -x + 2y \leq 6 \)
- \( 3x - 4y \leq 12 \)

Find the feasible region, marked as \( S \) and identify the corner points:

- \( (0,0) \)
- \( (4,0) \)
- \( (0,3) \)
- \( (2,3) \)
- \( (0,5) \)

Evaluate the objective function at each corner point:

- \( (0,0) \): \( R = 0 \)
- \( (4,0) \): \( R = 60 \)
- \( (0,3) \): \( R = 27 \)
- \( (3,3) \): \( R = 54 \)
- \( (0,5) \): \( R = 75 \)

The maximum occurs at \( (0,5) \): \( \frac{507}{7} \approx 72.43 \)

SOLN: \( \max R = \frac{507}{7} \) and it occurs at \( (\frac{3}{7}, \frac{3}{7}) \)
(b) Is there a minimum value of $R$ subject to these constraints?

yes, since $S$ is bounded a min exists.

From CP chart, we can see that the min value of $R = 0$ and it occurs at $(0,0)$. 
3. Solve the following linear programming (LP) problem:

(a) **OBJECTIVE:** Minimize \( C = 10x + 15y \)

**SUBJECT TO:**
\[
\begin{align*}
2x + 3y & \geq 12 \\
x - 3y & \leq 2 \\
x - y & \geq -3 \\
x & \geq 2 \\
y & \geq 2
\end{align*}
\]

**SOLN:**
- Min value of \( C = 60 \) and it occurs at every point on the line segment \( BC \).

**UNBOUNDED** in \( OI \) will be able to minimize our obj. func.

- \( A: (2, 5) \)
- \( B: (2, \frac{8}{3}) \)
- \( C: (3, 2) \)
- \( D: (8, 2) \)
(b) Is there a maximum value of $C$ subject to these constraints?

\[ \text{No, } S \text{ is unbounded in } \mathbb{Q}^1 \rightarrow \text{no max for } C=10x+15y \]
4. SET UP the following LP problem. DO NOT SOLVE.

A baker has 600 pounds of chocolate, 100 pounds of nuts, and 50 pounds of fruit, with which to make three types of candy, A, B, and C. A box of candy A uses 3 pounds of chocolate, 1 pound of nuts, 1 pound of fruit, and it sells for $8. A box of candy B requires 4 pounds of chocolate, 1/2 pound of nuts, and sells for $5. A box of candy C requires 5 pounds of chocolate, 3/4 pound of nuts, 1 pound of fruit, and sells for $6. How many boxes of each type of candy should be made from the inventory available in order to maximize revenue?

\[
\begin{align*}
A &= \text{# of boxes of candy A made/sold} \\
B &= \text{" } \\
C &= \text{" } \\
R &= \text{revenue (in $)} \\
\end{align*}
\]

**OBJ:** Max \( R = 8A + 5B + 6C \)

**SUBJECT TO:**
\[
\begin{align*}
3A + 4B + 5C &\leq 600 \text{ (lbs of choc)} \\
A + \frac{1}{2}B + \frac{3}{4}C &\leq 100 \text{ (lbs of nuts)} \\
A + C &\leq 50 \text{ (lbs of fruit)} \\
A \geq 0, B \geq 0, C \geq 0
\end{align*}
\]
5. SET UP the following LP problem. DO NOT SOLVE.

A rental company has two types of trucks; type A has 20 cu.ft. of refrigerated space and 40 cu.ft. of non-refrigerated space, and type B has 30 cu.ft. each of refrigerated and non-refrigerated space. A food plant must ship at least 900 cu.ft. of refrigerated produce and 1200 cu.ft. of non-refrigerated produce. If truck A rents for 30 cents per mile and truck B rents for 40 cents per mile, then how many of each truck should the plant rent in order to minimize rental costs?

\[
\begin{align*}
A &= \text{# of type A trucks} \\
B &= \text{# of type B trucks} \\
C &= \text{rental costs per mile (in \$)}
\end{align*}
\]

**OBJ:** Min \( C = 0.3A + 0.4B \)

**SUBJ TO:**
\[
\begin{align*}
20A + 30B &\geq 900 \text{ (cu.ft - refrig)} \\
40A + 30B &\geq 1200 \text{ (cu.ft - non-refrig)} \\
A &\geq 0, B \geq 0
\end{align*}
\]
6. A company has 100 kg of dry cereal and 125 kg of chocolate candy to be used in making two different snack mixes. One mix will contain half cereal and half candy, and will sell for $6 per kilogram. The other mix will contain 1/3 cereal and 2/3 candy, and will sell for $4.80 per kilogram. How many kilograms of each mix should the company prepare for maximum revenue? What is the maximum revenue?

**OBJ:** Max \( R = 6x + 4.80y \)

**SUBJ TO:**
- \( \frac{1}{2}x + \frac{1}{3}y \leq 100 \) (kg of cereal)
- \( \frac{1}{3}x + \frac{2}{3}y \leq 125 \) (kg of candy)

\( x \geq 0, y \geq 0 \)

\[ \frac{1}{2}x + \frac{1}{3}y \leq 100 \]
\[ x: \left( \frac{200}{1}, 0 \right) \]
\[ y: \left( 0, \frac{300}{1} \right) \]
\[ (0,0): 0 \leq 100 \]

\[ \frac{1}{3}x + \frac{2}{3}y \leq 125 \]
\[ x: \left( \frac{250}{1}, 0 \right) \]
\[ y: \left( 0, \frac{1875}{1} \right) \]
\[ (0,0): 0 \leq 125 \]

\[ x \geq 0, y \geq 0 \]

**Solution:**
Max revenue is $1260
When 150 kg of half/half mix + 75 kg of 1/3, 2/3 mix are made a sold.
7. A seamstress has 80 units of cotton material and 120 units of wool material. A suit requires 1 unit of cotton and 3 units of wool; a dress requires 2 units of each material. Due to previous demand, it has been determined that at most 10 suits should be made. If each suit brings in $20 profit and each dress brings in $30 profit, how many of each garment should be produced in order to maximize profit? Will there be any leftover resources when maximizing profit?

\[ \begin{align*}
\text{Suits made:} & \quad x = 0, y = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 \\
\text{Dresses:} & \quad y = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 \\
\text{Profit:} & \quad p = 20x + 30y \\
\text{Subject to:} & \quad x + 2y \leq 80 \quad \text{(units of cotton)} \\
& \quad 3x + 2y \leq 120 \quad \text{(units of wool)} \\
& \quad 0 \leq x \leq 10 \\
& \quad y \geq 0
\end{align*} \]
**CPs**

<table>
<thead>
<tr>
<th>A: (0,0)</th>
<th>B: (0,40)</th>
<th>C: (10,35)</th>
<th>D: (10,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1200</td>
<td>1250 *</td>
<td>200</td>
</tr>
</tbody>
</table>

**MAX**

**SOLN:** Max profit is $1250 when 10 suits + 35 dresses are made/sold.

**SUBJ TO**

\[ \begin{align*}
    x + 2y & \leq 80 \quad \text{(units of cotton)} \\
    3x + 2y & \leq 120 \quad \text{(units of wool)} \\
    0 & \leq x \leq 10 \\
    y & \geq 0
\end{align*} \]

**Avail**

- **Cotton:** \[ 10 + 2(35) = 80 \text{ units} \Rightarrow \text{no cotton leftover} \]
- **Wool:** \[ 3(10) + 2(35) = 100 \text{ units} \Rightarrow 20 \text{ units of wool leftover} \]