

Week-In-Review #8 (7.5, 7.6)

$$\underline{4} \cdot \underline{5} = 20 \text{ outcomes}$$

1. You roll a fair four-sided die (numbered 1-4) and a fair five-sided die (numbered 1-5).

(a) What is the probability that a 4 is rolled, given that two different numbers are rolled on the dice?

$$P(4 \mid \text{diff \#s}) = \frac{7}{16} \quad \leftarrow \begin{array}{l} \# \text{ w/ a 4 AND having diff \#s} \\ \# \text{ of outcomes w/ diff. \#s} \end{array}$$

(b) What is the probability that an even sum is rolled, if a number less than 4 is rolled on each of the dice?

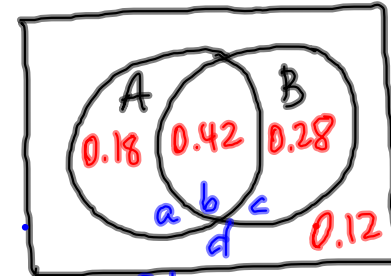
$$P(\text{even sum} \mid \underline{\underline{\# < 4 \text{ on both}}}) = \frac{5}{9}$$

<u>1</u> 1	<u>2</u> 1	<u>3</u> 1	<u>4</u> 1
<u>1</u> 2	<u>2</u> 2	<u>3</u> 2	<u>4</u> 2
<u>1</u> 3	<u>2</u> 3	<u>3</u> 3	<u>4</u> 3
<u>1</u> 4	<u>2</u> 4	<u>3</u> 4	<u>4</u> 4
15	25	35	45

2. You are given two events, A and B , where $P(A) = 0.6$, $P(B) = 0.7$, and $P(A^C \cap B^C) = 0.12$.

(a) Find $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{0.42}{0.7} = \boxed{0.6}$$



$$a + b + c + d = 1 \rightarrow c = 0.28$$

$$a + b = 0.6 \rightarrow a = 0.18$$

$$b + c + d = 0.7 \rightarrow b = 0.42$$

$$d = 0.12$$

(b) Are A and B mutually exclusive? Why or why not?

Mutually exclusive: no shared outcomes (disjoint)

If m.e. $\Rightarrow P(A \cup B) = P(A) + P(B)$
 $\Rightarrow P(A \cap B) = 0$

$P(A \cap B) = 0.42 \neq 0$
 \Rightarrow not mutually exclusive

(c) Are A and B independent? Why or why not?

Indep: knowing the outcome of an event doesn't change the prob. of another event occurring

If indep $\Rightarrow P(A \cap B) = P(A)P(B)$
 $\Rightarrow P(A|B) = P(A)$
 $\Leftrightarrow P(B|A) = P(B)$

$P(A \cap B) \stackrel{?}{=} P(A)P(B)$
 $0.42 \stackrel{?}{=} (0.6)(0.7)$
 $0.42 \stackrel{?}{=} 0.42 \Rightarrow$ Indep

3. The students in a statistics class were surveyed concerning their eye color. The findings were gathered in the table below.

	Blue Eyes (B)	Green Eyes (G)	Brown Eyes (W)	Other (O)	Totals
Females (F)	15	8	25	3	51
Males (M)	10	5	12	7	34
Totals	25	13	37	10	85

(a) Given a randomly selected student doesn't have brown eyes, what is the probability that the student is male?

$$P(M | W^c) = \frac{10+5+7}{25+13+10} = \frac{22}{48} \leftarrow \begin{array}{l} \# \text{ of males w/o brown eyes} \\ \# \text{ w/o brown eyes} \end{array}$$

(b) What is the probability that a randomly selected male does not have brown eyes?

$$P(W^c | M) = \frac{10+5+7}{34} \leftarrow \# \text{ of males} = \frac{22}{34}$$

(c) What is the probability that a randomly selected student is female if the student has blue eyes?

$$P(F | B) = \frac{15}{25}$$

3. The students in a statistics class were surveyed concerning their eye color. The findings were gathered in the table below.

	Blue Eyes (B)	Green Eyes (G)	Brown Eyes (W)	Other (O)	Totals
Females (F)	15	8	25	3	51 ←
Males (M)	10	5	12	7	34
Totals	25	13	37	10	85

(d) What is the probability that a randomly selected female has blue eyes?

$$P(B|F) = \frac{15}{51}$$

(e) What is the probability that a randomly selected student with green eyes is a male?

$$P(M|G) = \frac{5}{13}$$

(f) Are the events "student is female" and "student has blue eyes" mutually exclusive? Independent? (Why or why not?)

Mutually exclusive?

$$P(F \cap B) = \frac{15}{85} \neq 0$$

not mutually exclusive

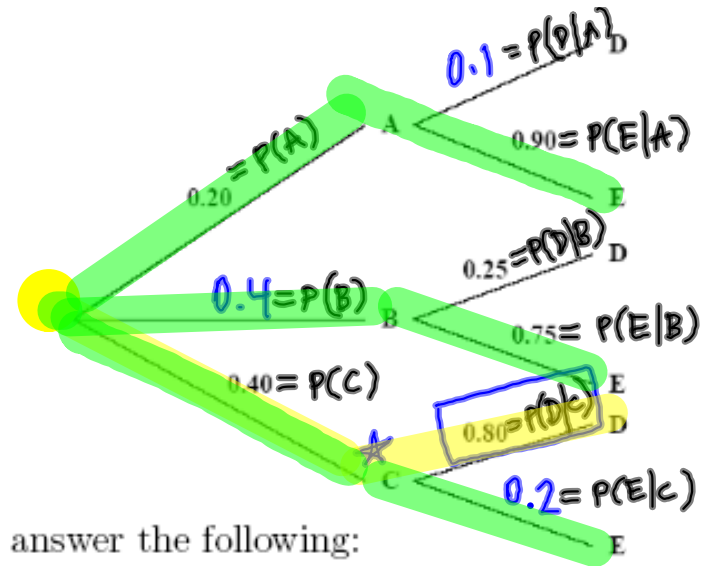
Indep.?

$$P(F \cap B) \stackrel{?}{=} P(F)P(B)$$

$$\frac{15}{85} \stackrel{?}{=} \left[\frac{51}{85} \right] \left[\frac{25}{85} \right]$$

$$\frac{3}{17} \stackrel{?}{=} \frac{3}{17} \quad \boxed{\text{Indep.}}$$

4.



Fill in the missing probabilities in the tree above and then answer the following:

(a) $P(A^C) = 1 - P(A) = 1 - 0.2 = \boxed{0.8}$

(b) $P(D|C) = \boxed{0.8}$ (# on tree) $\left(P(D|C) = \frac{P(D \cap C)}{P(C)} = \frac{0.4(0.8)}{0.4} = 0.8 \right)$

(c) $P(C \cap D) = \underbrace{P(C) \cdot P(D|C)}_{\text{product rule}} = 0.4(0.8) = \boxed{0.32}$ (one path thru tree)

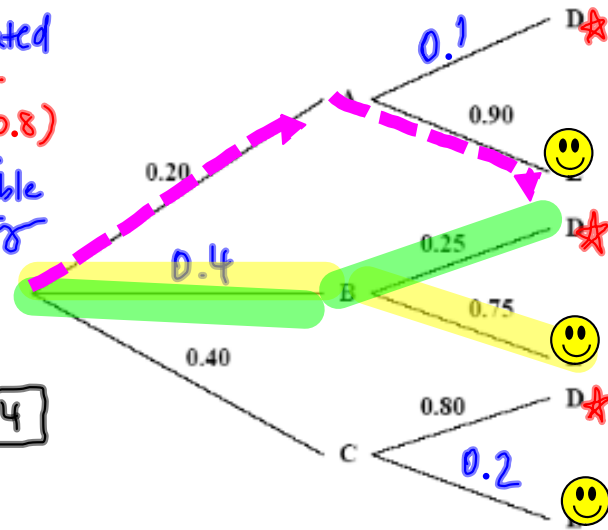
(d) $P(E) = P(A \cap E) + P(B \cap E) + P(C \cap E) = 0.2(0.9) + 0.4(0.75) + 0.4(0.2) = \boxed{0.56}$ (all paths that end at E)

(e) $P(E^C) = 1 - P(E) = 1 - \boxed{0.56} = \boxed{0.44}$ (same as $P(D)$)

$$(f) P(C | D^*) = \frac{P(C \cap D)}{P(D)} = \frac{0.4(0.8)}{0.2(0.1) + 0.4(0.25) + 0.4(0.8)} \leftarrow \text{path wanted}$$

$$= \frac{8}{11}$$

all possible paths ending at given



$$(g) P(B \cup D) = P(B) + P(D) - P(B \cap D)$$

$$= 0.4 + [0.44] - 0.4(0.25) = \boxed{0.74}$$

$$(h) P(B \cap D^c) = P(B \cap E) = 0.4(0.75) = \boxed{0.3}$$

$$(i) P(A \cup D^c) = P(A) + P(D^c) - P(A \cap D^c)$$

$$= 0.2 + [0.56] - 0.2(0.9)$$

$$= \boxed{0.58}$$

(j) Are A and E mutually exclusive? Independent?

Mutually Excl.

$$P(A \cap E) = 0.2(0.9)$$

$$= 0.18$$

$$\neq 0 \rightarrow \boxed{\text{not m.e.}}$$

Indep.?

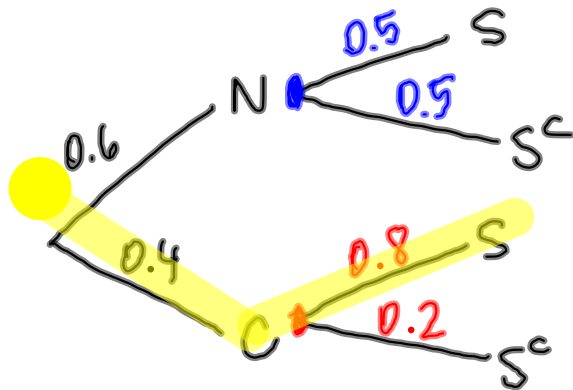
$$P(A \cap E) \stackrel{?}{=} P(A)P(E)$$

$$0.2(0.9) \stackrel{?}{=} [0.2][0.56]$$

Not indep. → Dependent

5. On Tuesday night, in a group of people, 60% watch shows on NBC and 40% watch shows on CBS. Of those who watch NBC, 50% watch TV at 7:00 PM and of those who watch CBS, 80% watch TV at 7:00 PM.

(a) Draw a tree representing the situation.



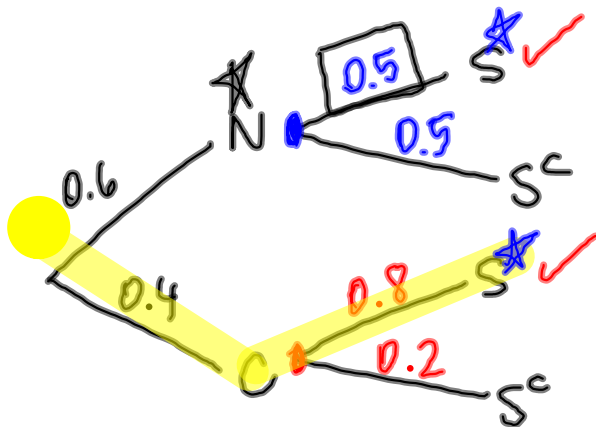
$N = \text{watch NBC}$
 $C = \text{ " CBS}$
 $S = \text{watches TV @ 7PM}$

(b) What's the probability that a randomly selected person from the group watches CBS on Tuesday night?

$$P(C) = \boxed{0.4}$$

(c) What's the probability that a randomly selected person watches CBS on Tuesday and watches TV at 7:00 PM?

$$P(C \cap S) = 0.4(0.8) = \boxed{0.32}$$



(d) What's the probability that a randomly selected person watches TV at 7:00 PM on Tuesday?

$$P(S) = 0.6(0.5) + 0.4(0.8) = \boxed{0.62}$$

(e) What's the probability that a Tuesday night NBC watcher from this group watched TV at 7:00 PM?

$$P(S | N) = \boxed{0.5}$$

(f) If you know a person from the group watches TV at 7:00 PM on Tuesday, is it more likely they are watching CBS or NBC?

$$P(N | S) = \frac{P(NS)}{P(S)} = \frac{0.6(0.5)}{[0.62]} = \frac{15}{31} \quad P(C | S) = \frac{P(CS)}{P(S)} = \frac{0.4(0.8)}{[0.62]} = \frac{16}{31} \text{ higher}$$

$\Rightarrow \boxed{\text{CBS}}$

6. Draw two cards in succession, without replacement, from a standard 52 card deck.

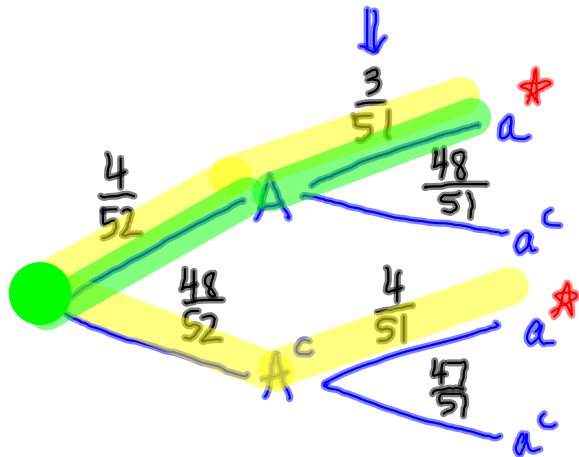
(a) Given that the first card drawn is an Ace, what is the probability that the second card drawn is an Ace?

$$P(2^{\text{nd}} \text{ Ace} \mid 1^{\text{st}} \text{ Ace}) = \frac{3}{51}$$

(b) What is the probability that the second card drawn is an Ace?

$$P(a) = \frac{4}{52} \left(\frac{3}{51} \right) + \frac{48}{52} \left(\frac{4}{51} \right) = \frac{1}{13}$$

(c) Given that the second card drawn is an Ace, what is the probability that the first card drawn is an Ace?

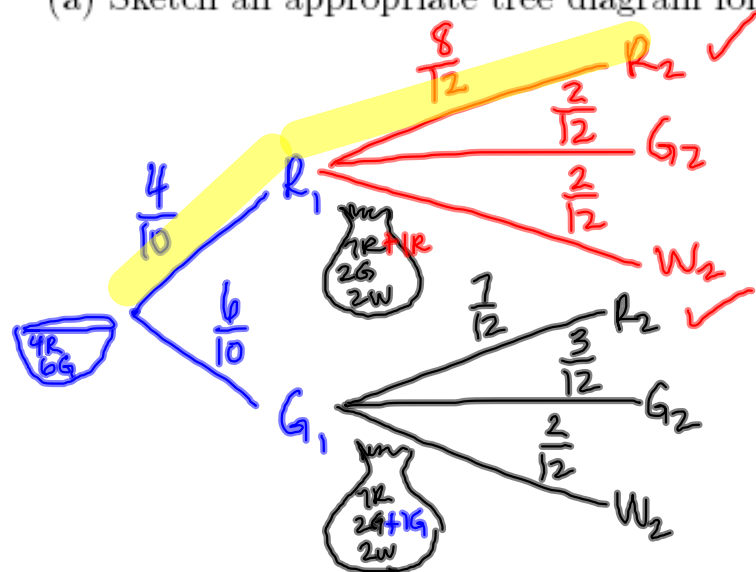


$$P(A \mid a^*) = \frac{P(A \cap a^*)}{P(a^*)} = \frac{\frac{4}{52} \left(\frac{3}{51} \right)}{\left[\frac{1}{13} \right]} = \frac{1}{17}$$

A = draw Ace on 1st
 a = " " " " 2nd

7. Consider the following experiment: Select a ball at random from a bowl (which has 4 red and 6 green balls) and place it into a bag (which has 7 red, 2 green, and 2 white balls). Then, draw a ball at random from the bag.

(a) Sketch an appropriate tree diagram for this experiment.



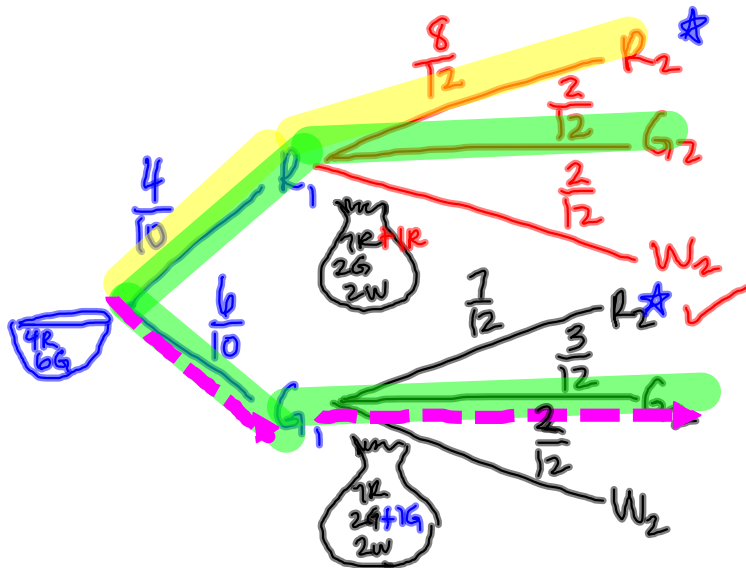
R_1 = red from bowl
 G_1 = green "
 R_2 = red from bag
 G_2 = green "
 W_2 = white "

(b) What is the probability that both balls drawn are red?

$$P(R_1 \cap R_2) = \frac{4}{10} \left(\frac{8}{12} \right) = \boxed{\frac{8}{30}}$$

(c) What is the probability that the second ball drawn is red?

$$P(R_2) = \frac{4}{10} \left(\frac{8}{12} \right) + \frac{6}{10} \left(\frac{7}{12} \right) = \boxed{\frac{37}{60}}$$



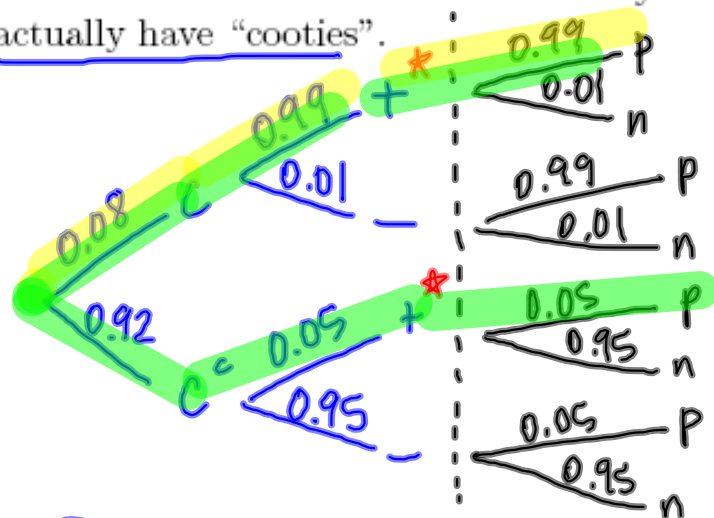
(d) If the second ball drawn is red, what is the probability that the first ball drawn is red?

$$P(R_1 | R_2^*) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\left[\frac{8}{30} \right]}{\left[\frac{37}{60} \right]} = \boxed{\frac{16}{37}}$$

(e) What is the probability that a green was drawn from the bowl or from the bag?

$$\begin{aligned} P(G_1 \cup G_2) &= P(G_1) + P(G_2) - P(G_1 \cap G_2) \\ &= \frac{6}{10} + \left\{ \frac{4}{10} \left(\frac{2}{12} \right) + \cancel{\frac{6}{10} \left(\frac{2}{12} \right)} \right\} - \cancel{\frac{6}{10} \left(\frac{2}{12} \right)} = \boxed{\frac{2}{3}} \end{aligned}$$

8. A test for "cooties" correctly indicates the presence of the condition 99% of the time among those who actually have "cooties". However, 5% of the time, the test indicates the presence of the condition in those who do not actually have it. It is known that 8% of those who take the test actually have "cooties".



$C = \text{has cooties}$
 $+/- = \text{test result}$
 $p = 2^{\text{nd}} \text{ test positive}$
 $n = \text{ " " negative}$

(a) If the test given to a person comes back positive, what is the probability that the person actually has "cooties"?

$$P(C | +) = \frac{P(C \cap +)}{P(+)} = \frac{0.08(0.99)}{0.08(0.99) + (0.92)(0.05)} = \boxed{\frac{198}{313}}$$

(b) What is the probability that a person actually has "cooties", given that they take the test twice and both times the test is positive for the condition?

$$P(C | + \cap p) = \frac{P(C \cap + \cap p)}{P(+ \cap p)} = \frac{0.08(0.99)(0.99)}{0.08(0.99)(0.99) + 0.92(0.05)(0.05)} \approx \boxed{0.9715}$$

9. There are 10 people in a class who each perform independently from one another on a test. If five people in the class each have an 80% chance of passing a test and the other five each have a 30% chance of passing the test, what's the probability that everyone in the class passes the test?

$N_i =$ the i^{th} person passes the test
all 10 people pass the test

$$P(N_1 \wedge N_2 \wedge N_3 \wedge N_4 \wedge N_5 \wedge N_6 \wedge N_7 \wedge N_8 \wedge N_9 \wedge N_{10})$$

$$= P(N_1)P(N_2)P(N_3)P(N_4)P(N_5)P(N_6)P(N_7)P(N_8)P(N_9)P(N_{10}) \quad \text{because indep!}$$

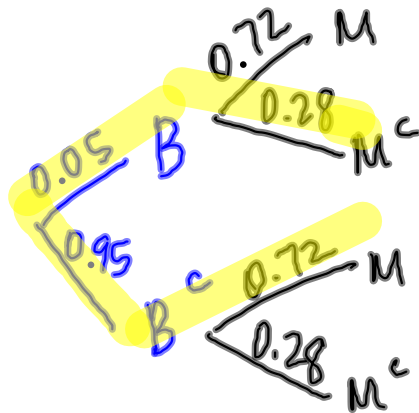
$$= (0.8)(0.8)(0.8)(0.8)(0.8)(0.3)(0.3)(0.3)(0.3)(0.3)$$

$$= (0.8)^5 (0.3)^5$$

$$\approx \boxed{0.0007963}$$

10. Ben and Michael each have a cell phone and are in the same math class. 5% of the time Ben forgets to turn off his phone before entering his math class and 72% of the time Michael forgets to turn his phone off before math class. If Ben and Michael have never spoken to each other and walk into class at different times, what is the probability that exactly one of them has his phone on during math class?

Ben + Michael indep. decide to turn off phones



$P(\text{exactly one on})$

$$\begin{aligned}
 &= P(B \cap M^c) + P(B^c \cap M) \\
 &= 0.05(0.28) + 0.95(0.72) \\
 &= \boxed{0.698}
 \end{aligned}$$

B = Ben has phone on in math class
 M = Michael " " " "