Exam II Review (Sections 4.1-4.5 and 5.1-5.6)

Note: This collection of questions is intended to be a brief overview of the exam material (with an emphasis at the beginning on material from Sections 5.5 and 5.6, which I have not previously reviewed). This is not intended to represent an actual exam. When studying you should also rework your notes, the previous week-in-reviews for Exam II material, and be familiar with your suggested and online homework problems.

1. Use the given graph to answer the following.

(a) If the graph is that of $f(x)$, find the value(s) of $x$, if any exist, where

(i) $f(x)$ has critical values $f' = 0$ or $f'$ DNE (in the domain of $f$)
\[ x = -3, 1, 3, 0 \]

(ii) $f(x)$ has a relative maximum $\uparrow$
\[ x = 0, 3 \]

(iii) $f(x)$ has a relative minimum $\downarrow$
\[ x = 1 \]

(iv) $f(x)$ has an absolute maximum (highest $y$-value)
\[ x = 3 \]

(v) $f(x)$ has an absolute minimum (lowest $y$-value)
DNE

(vi) $f(x)$ has an inflection point concavity changes
\[ x = -3, 2 \]
(b) If the graph is that of \( f'(x) \), find the interval(s) or value(s) of \( x \), if any exist, where

(i) \( f(x) \) is increasing

\[ f' > 0 \quad (f' \text{ above x-axis}) \]

\[ (2, 4) \]

(ii) \( f(x) \) is concave down

\[ f'' < 0 \quad \Rightarrow \quad f' \text{ decreasing} \]

\[ (0,1) \cup (3, \infty) \]

(iii) \( f(x) \) has a critical value (Are any relative extrema of \( f(x) \)?)

\[ f' = 0 \text{ or } f' \text{ DNE (in the domain of } f) \]

\[ \downarrow \text{ int. of } f' \]

\[ x \text{-values not in domain of this graph} \]

\[ x = 0, 2, 4 \]

(iv) \( f(x) \) has a point of inflection

\[ f'' + - \]

\[ f' \text{ max } \]

\[ f \text{ max} \]

\[ f' \text{ min } \]

\[ f \text{ min} \]

\[ x = 0, 1, 3 \]
(c) If the graph is that of \( f''(x) \), find the interval(s) or value(s) of \( x \), if any exist, where

(i) \( f'(x) \) is decreasing \( \rightarrow f'' < 0 \) (\( f'' \) below \( x \)-axis)

\((-\infty, 0) \cup (0, 2) \cup (4, \infty)\)

(ii) \( f(x) \) is concave up \( \rightarrow f'' > 0 \) (\( f'' \) above \( x \)-axis)

\((2, 4)\)

(iii) \( f(x) \) has a point of inflection \( \rightarrow f'' \) changes sign \( \Rightarrow f'' + - or - + \)

\( \rightarrow f'' \) crosses \( x \)-axis

\( x = 2, 4 \)
2. Find the absolute extrema of \( f(x) = \frac{x^2}{x+2} \), if any exist, over the following intervals.

\[ f'(x) = \frac{(x+2)(2x) - x^2(1)}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2} = \frac{x^2 + 4x}{(x+2)^2} = \frac{(x)(x+4)}{(x+2)^2} \]

CVs:
- \( f' = 0 \) → \( x = 0, x = -4 \)
- \( f' \) DNE → \( x = -2 \) (not in D, not a CV)

(a) \([-10, -3]\) (closed interval)
- \( f \) cont. on this interval
- \( f(x) = \frac{x^2}{x+2} \)
- \( x \) \| \( f(x) \)
  - \(-10\) \| \(-12.5 \leq \text{ABS MIN} \)
  - \(-4\) \| \(-8 \leq \text{ABS MAX} \)

(b) \((-1, \infty)\) (open interval)
- Only one CV (\( x = 0 \)) in the interval
- \( f \) cont. on interval
- \( f'(x) \)
  - \( x \) \| \( f'(x) \)
  - \(-1 \) \( \downarrow \) \( 0 \) \( \uparrow \) \( \infty \)
  - \( f''(x) \)
    - \( f''(0) = \frac{0}{(0+2)^3} = 0 \) at \( x = 0 \)
    - No ABS MAX

(c) \((\infty, \infty)\)
- \( f \) not cont. on this interval
- Look at graph
- No ABS MIN
- No ABS MAX

\[ f''(x) = \frac{(x+2)^3 [2x + 4] - (x^2 + 4x)(2(x+2))(1)}{(x+2)^4} = \frac{8}{(x+2)^3} \]

\[ f''(0) = \frac{0}{(0+2)^3} = 0 \] at \( x = 0 \)
3. From a 27-inch by 9-inch piece of cardboard, square corners are cut out so that the sides can be folded up to form a box with no top. Use calculus to determine the maximum volume the box will be able to hold.

\[ \text{Max } V = LWH = (9-2x)(27-2x)(x) = (243-18x-54x+4x^2)x \]

\[ \text{Max } V = 4x^3 - 72x^2 + 243x \]

- \( V \) is cont. everywhere
- \( V' = 12x^2 - 144x + 243 \)
- CVs: \( V' = 0 \): \( 12x^2 - 144x + 243 = 0 \)
  \[ x = \frac{144 \pm \sqrt{(-144)^2 - 4(12)(243)}}{2(12)} \]
  \[ = \frac{144 \pm \sqrt{9072}}{24} \approx 9.9666, 20.314 \]

\[ V'' = 24x - 144 \]

V' never equal to 0 in \((0, 9/2)\) interval for x

V''(20314) = -144 < 0

\( V \) is a max at \( x \approx 9.9666 \)

\[ \text{Max } V \approx 230.470 \text{ in}^3 \]
4. A farmer has 1200 m of fencing. He wants to enclose a rectangular field bordering a river, with no fencing needed along the river. Use calculus to answer the following questions.

(a) Find the fence dimensions that will maximize the enclosed area.

(b) What is the maximum enclosed area?

\[ x + y + x = 1200 \]
\[ 2x + y = 1200 \]
\[ y = 1200 - 2x \]

\[ \text{Max } A = xy \]
\[ = x(1200 - 2x) \]
\[ \text{Max } A = 1200x - 2x^2 \]

\[ A' = 1200 - 4x \]

\[ \text{CV is a max} \]
\[ A''(300) = 0 \]

\[ A' = 0 \]
\[ 1200 - 4x = 0 \]
\[ 4x = 1200 \]
\[ x = 300 \]

\[ \text{only one CV in interval} \]

\[ A'' = -4 \]

\[ \text{Dim: } x = 300 \ m \]
\[ y = 1200 - 2(300) = 600 \ m \]

\[ \text{Max Area} = xy = 300(600) = 180,000 \text{ m}^2 \]
5. A bus company charged $60 per person for a sight-seeing trip and obtained 40 people for the trip. The company has data that indicates that for the same trip, each $2 increase in the price above $60 results in a loss of one customer. The bus company has fixed costs of $3000 for each trip and additional costs of $4 per customer. Use calculus to answer the following questions.

\( x = \# \text{ of customers} \)

(a) What should the company charge in order to maximize revenue?

\[
\text{Rev} = \text{(price)} \times \text{(quantity)} = (\text{demand}) (x) = (-2x + 140)(x) = -2x^2 + 140x
\]

\[
\Delta x = -1 \quad \Delta p = +2
\]

\[
\frac{\Delta p}{\Delta x} = \frac{2}{-1} = -2
\]

\[
p - 60 = -2(x - 40)
\]

\[
p - 60 = -2x + 80
\]

\[
p = -2x + 140
\]

\[
\text{Demand:} \quad (x, p)
\]

\[
\Delta x = -1 \quad \Delta p = +2
\]

\[
\frac{\Delta p}{\Delta x} = \frac{2}{-1} = -2
\]

\[
p - 60 = -2(x - 40)
\]

\[
p - 60 = -2x + 80
\]

\[
p = -2x + 140
\]

\[
\text{Interval}
\]

\[
\frac{x \geq 0 \quad p \geq 0}{-2x + 140 \geq 0}
\]

\[
-2x \geq -140
\]

\[
x \leq 70
\]

\[
\text{Max} \quad R = -2x^2 + 140x \quad \text{on} \quad [0, 70]
\]

\[
R \text{ cont everywhere}
\]

\[
R' = -4x + 140
\]

CVS: \( R' = 0 \) \quad -4x + 140 = 0 \quad 4x = 140 \quad x = 35 \quad \text{in interval}

\[
\text{price} = -2x + 140
\]

\[
\text{Max Rev} \quad 35 \quad 2450
\]

\[
70 \quad 0
\]

\[
\text{Revenue} = \boxed{70}
\]
(b) What should the company charge in order to maximize profits?

\[ P = R - C \]
\[ = (-2x^2 + 140x) - (4x + 3000) \]
\[ \text{Max } \rho = -2x^2 + 136x - 3000 \text{ on } [0, 70] \]

\[ P \text{ cont everywhere} \]
\[ P' = -4x + 136 \]

**CNS:**

\[ P' = 0 \Rightarrow -4x + 136 = 0 \]
\[ 4x = 136 \]
\[ x = 34 \]

(in our interval)

\[ \begin{array}{c|c|c|c}
 x & P & X & \text{Profit} \\
\hline
 0 & -3000 & 0 & \text{Not profitable business} \\
 34 & -688 & \text{MAX PROFIT} & \text{(Not profitable business)} \\
 70 & -3280 & & \\
\end{array} \]

\[ \text{Price } = -2x + 140 \]
\[ = -2(34) + 140 \]
\[ = 72 \]
6. Find the first derivative of the following functions. (DO NOT SIMPLIFY.)

(a) \( f(x) = x^3 - \frac{1}{x^2} + 2\sqrt{x} - e^2 + 4 = x^3 - x^{-2} + 2x^{1/2} - e^2 + 4 \)

\[ f'(x) = 3x^2 + 2x^{-3} + x^{-1/2} - 0 + 0 \]

(b) \( f(x) = \sqrt[4]{x^2 + 6x + 1} \)

\[ f'(x) = 4 \left( x^2 + 6x + 1 \right)^3 \left( 2x + 6 \right) \]

(c) \( f(x) = (2x + 1)\sqrt{x^2 + 1} = (2x + 1) \left( x^2 + 1 \right)^{1/2} \)

\[ f'(x) = \left[ 2 \right] \left( x^2 + 1 \right)^{1/2} + (2x + 1) \left[ \frac{1}{2} \left( x^2 + 1 \right)^{-1/2} (2x) \right] \]
(d) \( f(x) = \frac{3}{\sqrt{0.5x^7 + 9}} = 3 \left( 0.5x^7 + 9 \right)^{-\frac{1}{2}} \)

\[ f'(x) = -\frac{3}{5} \left( 0.5x^7 + 9 \right)^{-\frac{6}{5}} (3.5x^6) \]

(e) \( f(x) = \frac{|x+1|}{(x-2)^3} \)

\[ \text{quo. rule} \quad f'(x) = \frac{(x-2)^3 \left[ 1 \right] - (x+1) \left[ 3(x-2)^2 \cdot 1 \right]}{(x-2)^3} \]

(f) \( f(x) = \sqrt[4]{3x^2(2x - 8x^6)^4} = \left[ 3x^2(2x - 8x^6)^4 \right]^{\frac{1}{4}} = (3x^2)^{\frac{1}{4}}(2x - 8x^6)^{4/3} \)

\[ \text{prod. rule} \quad f'(x) = \left[ \frac{1}{3} \left( \frac{3x^2}{2x - 8x^6} \right)^{\frac{3}{4}} \right] (2x - 8x^6)^{4/3} + (3x^2)^{\frac{1}{3}} \left[ \frac{5}{3} \left( 2x - 8x^6 \right)^{1/3} (2 - 48x^5) \right] \]
(g) \( y = 23^{x^2 + 4x} + 3^2 \)

\[ y' = 23^{x^2 + 4x} \cdot (2x + 4)(\ln 23) + 0 \]

(h) \( y = \log_{10}(\ln(9-x)) + \ln 5 \)

\[ y' = \left( \frac{1}{\ln 10} \right) \left( \frac{1}{\ln(9-x)} \right) \left( -\frac{1}{9-x} \right) + 0 \]

\[ \square = \ln \left( \frac{9-x}{x} \right) \]
\[ \square' = \frac{-1}{9-x} \]

7. Find \( \frac{dy}{dx} \) given \( y = e^u \) and \( u = -2x^2 \)

\[ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \]

\[ = (e^u)(-4x) \]

\[ \frac{dy}{dx} = e^{-2x^2}(-4x) \]

1st get \( y \) as a function of \( x \)

\[ y = e^{-2x^2} \]

\[ \frac{dy}{dx} = e^{-2x^2}(-4x) \]

\[ \frac{dy}{dx} = e^{-2x^2}(-4x) \]
8. Find the instantaneous rate of change of \( f(x) = \sqrt{x+7}^3 \) at \( x = 0 \).

\[
f'(0) = ?
\]

\[
f = (x+7)^{\frac{3}{2}}
\]

\[
f' = \frac{3}{2} (x+7)^{\frac{1}{2}} (1)
\]

\[
f'(0) = \frac{3}{2} (0+7)^{\frac{1}{2}} = \frac{3}{2} \sqrt{7}
\]

9. Find the equation of the line tangent to \( f(x) = \ln \left( \frac{x^4}{x^2+1} \right) \) at \( x = 1 \).

\[
\text{pt: } (1, f(1)) = (1, \ln \left( \frac{1}{2} \right))
\]

\[
m = f'(1) = 3
\]

\[
y - \ln \left( \frac{1}{2} \right) = 3 (x - 1)
\]
10. If $C(x) = x^2 - x + 400$ is the cost of producing $x$ items, in dollars, and $p = -3x + 200$ is the price-demand function for the items, find the following:

(a) $R(x) = p \cdot x = (-3x + 200)(x) = -3x^2 + 200x$

(b) $P(x) = R - C = (-3x^2 + 200x) - (x^2 - x + 400)$
   
   $P(x) = -4x^2 + 201x - 400$

(c) Marginal profit function
   
   $P'(x) = -8x + 201$

(d) The approximate profit from selling the 11th item
   
   $P'(10) = -8(10) + 201 = \$121$

(e) The exact profit from selling the 20th item
   
   $P(20) - P(19) = \$45$
11. If \( C(x) \) represents a company’s cost function, what does \( C'(5) = 90 \) represent?

When making 5 items, the co’s costs are increasing by \( \$90/\text{item} \).

OK

The approximate cost of making the 6th item is \( \$90 \).

12. The pulse rate (in number of heartbeats per minute) of a long distance runner \( t \) seconds after leaving the starting line is given by

\[
P(t) = \frac{300\sqrt{0.5t^2 + 2t + 25}}{t + 25} = \frac{300 \left(0.5t^2 + 2t + 25\right)^{\frac{1}{2}}}{t + 25}
\]

(a) Find \( P(120) \) and interpret your answer.

\[
P(120) \approx 179 \quad \rightarrow \quad 120 \text{ sec. after leaving the starting line, a long dist. runner's pulse is } \approx 179 \text{ beats/min.}
\]

(b) Find \( P'(120) \) and interpret your answer.

\[
P'(t) = \frac{(t+25)\left[150(0.5t^2 + 2t + 25)^{\frac{1}{2}}(t+2)\right] - 300(0.5t^2 + 2t + 25)^{\frac{1}{2}}(1)}{(t+25)^2}
\]

\[
P'(120) \approx -0.69 \quad \rightarrow \quad \text{after 120 sec, the runner's pulse is decreasing by } \approx 0.69 \text{ (beats/min)/sec}
\]
13. Given the price-demand equation \( x = 10(p - 9)^2 \), make sure demand is a func of price.

(a) Find the elasticity of demand, \( E(p) \).

\[
E(p) = -p \cdot \frac{d(x)}{d(p)} = -p \cdot \frac{x'}{x} = -p \left[ \frac{2}{10(p-9)x} \right] = \frac{-2p}{p-9}
\]

(b) If \( p = $5 \), is demand elastic, inelastic, or is there unit elasticity?

\[
E(5) = \frac{-2(5)}{5-9} = \frac{-10}{-4} = 2.5 \rightarrow \text{elastic}
\]

(c) At a price of $5, if the price is increased by 2%, what is the approximate percentage change in demand?

\[
\% \Delta \text{ in demand} = E(p)(\% \text{ change in price}) = (2.5)(2) = 5 \% \text{ decrease in demand}
\]

(d) Should the price be raised, lowered, or kept at $5 in order to increase revenue?

\( \text{lower} \)

(e) At what price will revenue be maximized?

\[
E(p) = 1 \Rightarrow \frac{-2p}{p-9} = 1
\]

\[
-2p = p - 9 \rightarrow -3p = -9 \rightarrow p = \#3
\]
14. Find the domain, all asymptotes, intercepts and any "holes" which occur in the graph of the following function:

\[ f(x) = \frac{4(x - 3)(x - 1)}{(2x - 2)(x + 4)} = \frac{4(x - 3)(x - 1)}{2(x - 1)(x + 4)} \]

**Domain:** \( x \neq 1, -4 \) \( (-\infty, -4) \cup (-4, 1) \cup (1, \infty) \)

**VA/holes**

look where denom = 0 \( [x=1,-4] \)

**VA:** After simplification, x-value is still a problem in denom \( x = -4 \)

**Hole:** After simplification, x-value is not a problem in denom \( x = 1 \)

**Intercepts**

\( x\text{-int (} y = 0 \text{)} \rightarrow 0 = 4(x - 3) \)

\( x = 3 \quad \) \((3, 0)\)

\( y\text{-int (} x = 0 \text{)} : f(0) = \frac{4(0 - 3)}{2(0 + 4)} = \frac{-12}{8} = -\frac{3}{2} \quad \) \((0, -\frac{3}{2})\)

\[ \frac{4(x^2 - 4x + 3)}{2x^2 + 8x - 2x - 8} = \frac{4x^2 - 16x + 12}{2x^2 - 6x - 8} \]

\[ \text{HA} \quad \left( \lim_{x \to \pm \infty} f(x) \right) \]

\[ y = \frac{4}{2} = 2 \]

\[ \lim_{x \to \infty} \frac{4x^2 - 16x + 12}{2x^2 - 6x - 8} \]

\[ = \lim_{x \to \infty} \frac{4 - \frac{16}{x} + \frac{12}{x^2}}{2 + \frac{6}{x} - \frac{8}{x^2}} = \frac{4}{2} = 2 \]

\( \text{HA: } y = 2 \)
15. Find the interval(s) where \( f(x) = e^{x^3 + 2x^2 - 4x} \) is increasing/decreasing and determine any points of relative extrema.

\[
D_f: \mathbb{R} \\
\dot{f} = e^{x^3 + 2x^2 - 4x} \left( 3x^2 + 4x - 4 \right) = e^{x^3 + 2x^2 - 4x} \left( 3x^2 + 4x - 4 \right)
\]

CVs: \( f' \) def. everywhere

\[
\dot{f} = 0 ? \quad e^{x^3 + 2x^2 - 4x} \left( 3x^2 + 4x - 4 \right) = 0 \\
e^{x^3 + 2x^2 - 4x} \neq 0 \\
3x^2 + 4x - 4 = 0 \\
(3x - 2)(x + 2) = 0 \\
3x - 2 = 0 \quad x + 2 = 0 \\
x = \frac{2}{3} \quad x = -2
\]

Inc: \((-\infty, -2) \cup \left( \frac{2}{3}, \infty \right)\)
Dec: \((-2, \frac{2}{3})\)

Rel Max: \((-2, f(-2))\)
Rel Min: \(\left( \frac{2}{3}, f\left( \frac{2}{3} \right) \right)\)
16. Find intervals, if any exist, where \( f(x) = \ln(x^2) \) is concave up/concave down and determine any points of inflection.

\[
D: \quad x^2 > 0 \quad \text{always true when } x \neq 0
\]

\[
D: \quad x \neq 0
\]

\[
f'(x) = \frac{2x}{x^2} = \frac{2}{x} = 2x^{-1}
\]

\[
f''(x) = -2x^{-2} = \frac{-2}{x^2}
\]

\[
f''(x) = 0 \quad \text{if } x = 0
\]

\[
f''(x) \text{ DNE } \quad \text{if } x = 0 \quad (\text{not in } D)
\]

\[
\text{CI: } (-\infty, 0) \cup (0, \infty)
\]

\[
\text{CT: nowhere}
\]

\[
\text{IF: none}
\]
17. Sketch the graph of a function that satisfies the following conditions:

- **x-intercepts**: (1, 0) and (-2, 0)
- **Vertical Asymptotes** at \( x = -1, 2 \)
- \( \lim_{x \to -\infty} f(x) = 0 \) \( \implies \) HA: \( y = 0 \) (both directions)
- \( f(-5) = 1, f(4) = -1 \) \( \implies \) pts on graph

\[ f'(x) > 0 \text{ on } (-\infty, -4), (1, 2) \text{ and } (2, \infty) \]
\[ f'(x) < 0 \text{ on } (-4, -1) \text{ and } (-1, 1) \]
\[ f''(x) > 0 \text{ on } (-\infty, -6) \text{ and } (-1, 2) \]
\[ f''(x) < 0 \text{ on } (-6, -1) \text{ and } (2, \infty) \]
18. Fill in the blanks below.

- $f''(x) > 0$ means that
  the graph of $f''(x)$ is above x-axis, $f'(x)$ is increasing,
  and $f(x)$ is concave up.

- $f''(x) < 0$ means that
  the graph of $f''(x)$ is below x-axis, $f'(x)$ is decreasing,
  and $f(x)$ is concave down.

- $f'(x) > 0$ means that
  the graph of $f'(x)$ is above the x-axis and $f(x)$ is increasing.

- $f'(x) < 0$ means that
  the graph of $f'(x)$ is below the x-axis and $f(x)$ is decreasing.

- $x$-intercepts of $f'(x)$ appear as horizontal tangents in the graph of $f(x)$.
  $\frac{f'}{0}$

- Relative extrema of $f'(x)$ appear as inflection pts in the graph of $f(x)$. 
19. If \( f(x) \) is a continuous function where \( f'(2) = 0 \) and \( f''(2) = 8 \), what (if anything) can be concluded about the behavior of \( f(x) \) at \( x = 2 \)?

\[
\begin{align*}
\quad f'(2) &= 0 \quad \text{(cv)} \\
\quad f''(2) &= 8 \Rightarrow f''(2) \quad \sqrt{f} \quad f' = 0 \\
\quad \text{At } x = 2, \text{ rel. min.}
\end{align*}
\]

20. Describe the end behavior of \( f(x) = \sqrt{a^2x^7 - bx^5 + cx - 100} \), if \( a, b, \) and \( c \) are all negative constants.

\( f(x) \) is an odd degree \( -7 \) polynomial

lead. Coeff = \( a^2 = \text{positive} \)

\( (a \text{ neg} \Rightarrow a^2 \text{ positive}) \)

\[
\begin{align*}
\lim_{x \to -\infty} f(x) &= -\infty \\
\lim_{x \to +\infty} f(x) &= +\infty
\end{align*}
\]
21. Find each of the following limits and explain what the limit result means for the graph of the function.

(a) \( \lim_{x \to \infty} \frac{3x^4 - x + x^3}{4x^3 - x^2 + 5x^4} = \frac{3}{5} \quad \rightarrow \quad \text{HA: } y = \frac{3}{5} \)

\[
\lim_{x \to \infty} \frac{3x^4}{x^4} - \frac{x}{x^4} + \frac{x^3}{x^4} = \lim_{x \to \infty} \frac{3 - \frac{1}{x^3} + \frac{1}{x^2}}{\frac{4x^3}{x^4} - \frac{x^2}{x^4} + 5\frac{x^4}{x^4}} = \frac{3}{5}
\]

(b) \( \lim_{x \to -\infty} \frac{2x^2 - 1}{-3x - 6} \)

\( \text{no HA} \)

\[
\lim_{x \to \infty} \frac{\frac{2x^2}{x} + \frac{1}{x}}{\frac{-3x}{x} - \frac{6}{x}} = \lim_{x \to \infty} \frac{2x + \frac{1}{x}^0}{-3 - \frac{6}{x}^0} = \frac{\infty}{-3} \quad \rightarrow \quad \square
\]

\( \downarrow \quad \text{No HA} \)
(c) \[ \lim_{x \to \infty} \frac{\frac{x^3}{x^4} + \frac{\frac{x^2}{x^4}}{2}} = 0 \rightarrow \text{HA: } y = 0 \]

\[ \lim_{x \to \infty} \frac{\frac{x^3}{x^4} + \frac{\frac{x^2}{x^4}}{2}} = 0 \rightarrow \text{HA: } y = 0 \]

(d) \[ \lim_{x \to \infty} \frac{5e^x - 1}{e^{-x} + 2e^x} = \frac{\infty - 1}{0 + \infty} = \frac{\infty}{\infty} \]

\[ \lim_{x \to \infty} \frac{5e^x}{e^x + 2e^x} = \lim_{x \to \infty} \frac{5 - e^{-x}}{e^{2x} + 2} = \frac{5}{2} \rightarrow \text{HA: } y = \frac{5}{2} \text{ (on right)} \]