

EXTRA COUNTING PROBLEMS

1. **A box of 42 batteries contains 13 defective batteries. If you randomly select a sample of 9 batteries, how many samples have at least one defective battery?**

Total number of samples – number of samples with 0 defective batteries (all 9 non-defective)

$$C(42, 9) - C(13, 0)C(29, 9) = 435,876,805$$

2. **Bob and Jan go to the movies with 6 of their friends. They all sit in the front row, which has exactly 8 chairs. How many different arrangements of these 8 people are possible if Bob and Jan must sit next to each other?**

We must seat Bob and Jan together and then seat their friends in the remaining seats.

There are 7 pairs of seats in the row of 8 chairs, in which Bob and Jan could choose to sit. Within these 7 pairs, there are $2! = 2$ ways for Bob and Jan to physically sit in the pairs of seats. Thus, there are $7 \cdot 2! = 14$ ways for Bob and Jan to sit together.

There will be six remaining seats for their friends and there are $6!$ ways to seat 6 people in 6 seats.

Therefore, there are a total of $7 \cdot 2! \cdot 6! = 10,080$ ways for all 8 to sit in the front row, with Bob and Jan together.

3. **How many license plates with 3 letters and 3 digits can be formed if the letters must be kept together, digits must be kept together, no letters can be repeated, and each of the 3 digits is odd?**

Since there are 3 letters and 3 digits, and these must be kept together, there are only 2 ways to do this: LLLDDD or DDDLLL

Letters must all be different: $\underline{26} \cdot \underline{25} \cdot \underline{24}$ Digits must all be odd: $\underline{5} \cdot \underline{5} \cdot \underline{5}$

$$2 \cdot (\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{5} \cdot \underline{5} \cdot \underline{5}) = 3,900,000$$

4. **In how many ways can a committee with 19 members elect a chair, vice chair, secretary, treasurer, and a subcommittee of 5 members, if officers cannot serve on the subcommittee?**

Specific titles means order matters for these positions. For the subcommittee, the order of the 5 members doesn't matter.

$$\text{Method 1: } P(19, 4) \cdot C(15, 5) = 279,351,072$$

$$\text{Method 2: } \underline{19} \cdot \underline{18} \cdot \underline{17} \cdot \underline{16} \cdot C(15, 5) = 279,351,072$$

5. A sample of 6 disks is drawn from a bag containing 3 green, 4 red, 5 orange, and 2 blue disks. In how many ways can a sample with

- (a) **At most 1 red be drawn?**

At most 1 red means exactly 0 red OR exactly 1 red. Since these are disjoint cases, we can add up the number of ways for each case.

$$\begin{array}{rcl} 0 \text{ red and 6 not red} & \text{OR} & 1 \text{ red and 5 not red} \\ C(4,0)C(10,6) & + & C(4,1)C(10,5) \end{array} = 1218$$

- (b) **Exactly 3 green or exactly 3 orange be drawn?**

Here there are two cases, but they are not disjoint since we could have 3 green and 3 orange at the same time in a sample. So, we must add up the separate cases and then subtract the number of ways to have both at the same time to avoid double counting.

$$\begin{array}{rcl} n(3 \text{ green} \cup 3 \text{ orange}) & & \\ = & n(3 \text{ green}) & + \quad n(3 \text{ orange}) \quad - \quad n(3 \text{ green} \cap 3 \text{ orange}) \\ = & 3 \text{ green and 3 not green} & \quad 3 \text{ orange and 3 not orange} \quad \quad 3 \text{ green and 3 orange} \\ = & C(3,3)C(11,3) & + \quad C(5,3)C(9,3) \quad - \quad C(3,3)C(5,3) \quad = 995 \end{array}$$

6. A child has a set of plastic objects that includes 4 pyramids, 6 cubes, and 3 spheres. How many ways can the child place all of the objects in a row if objects of the same type are identical?

$$\frac{13!}{4! \cdot 6! \cdot 3!} = 60,060$$

7. You have 10 DVDs on a shelf: 5 dramas, 3 comedies, and 2 action movies. In how many ways can you arrange these DVDs so that all DVDs of the same genre are grouped together?

Arrange the order of the 3 groups and then arrange the items in each group.

$$3! \cdot (5! \cdot 3! \cdot 2!) = 8,640$$