

COUNTING EXERCISES

1. A high school club has 20 total members (8 seniors, 7 juniors and 5 sophomores).

- (a) In how many different ways can a President, VP, and Secretary be chosen for the club, if the President must be a senior, the VP a junior and the Secretary a sophomore?

$$\frac{8}{P} \cdot \frac{7}{VP} \cdot \frac{5}{S} = \boxed{280}$$

- (b) In how many different ways can a President, VP, and Secretary be chosen for the club, if the President must be a senior and the VP and the Secretary must be a junior or a sophomore?

$$\frac{8}{P} \cdot \frac{12}{VP} \cdot \frac{11}{S} = \boxed{1056}$$

- (c) In how many different ways can all the members be arranged in a single row for a yearbook photo?

$$\boxed{20!} \approx 2.4329 \times 10^{18}$$

- (d) How many different yearbook photos can be taken where all the seniors are in a row behind a row of all the remaining club members?

$$\left[\frac{8!}{S} \cdot \frac{12!}{\text{others}} \right] \approx 1.9313 \times 10^{13}$$

- (e) How many different one row yearbook photos can be taken if only the first 10 club members present are allowed in the photo?

$$\boxed{P(20, 10)} \approx 6.7044 \times 10^{11}$$

- (f) How many different photos are possible if the condition in part (e) must be met, and it is known the President of the club will be one of the 10 and must be standing on one end of the row?

$$2 \times \frac{1}{P} \cdot \frac{19}{19} \cdot \frac{18}{18} \cdot \frac{17}{17} \cdot \frac{16}{16} \cdot \frac{15}{15} \cdot \frac{14}{14} \cdot \frac{13}{13} \cdot \frac{12}{12} \cdot \frac{11}{11} = \frac{2 \cdot P(19, 9)}{P \cdot \text{others}} \approx 6.7044 \times 10^{10}$$

- (g) How many different one row yearbook photos can be taken if all members of the same classification must be standing next to one another?

$$\boxed{3! (8! 7! 5!)} \approx 1.4631 \times 10^{11}$$

- (h) In how many different ways can a group of 5 be selected from the members of this club?

$$C(20, 5) = \boxed{15,504}$$

- (i) In how many different ways can two groups of 5 be selected from the members of this club?

$$C(20, 5) C(15, 5) = \boxed{46,558,512}$$

2. A secretary has a cup on her desk that is filled with 4 wooden pencils, 10 pens, and 2 mechanical pencils. She randomly selects 6 writing utensils from the cup without looking.

16 Total → Pick 6
4 WP
10 pens
2 MP

(a) How many different groups of writing utensils can she grab?

$$C(16, 6) = \boxed{8008}$$

(b) How many different groups with exactly 3 pens can she grab?

$$\frac{C(10, 3)}{3 \text{ pens}} \cdot \frac{C(6, 3)}{3 \text{ not pens}} = \boxed{2400}$$

(c) How many different groups with exactly 2 pens and exactly 3 wooden pencils can she grab?

$$\frac{C(10, 2)}{2 \text{ pens}} \cdot \frac{C(4, 3)}{3 \text{ WP}} \cdot \frac{C(2, 1)}{1 \text{ MP}} = \boxed{360}$$

(d) How many different groups with at least one pencil can she grab?

$$\frac{C(16, 6)}{\text{Total}} - \frac{C(10, 6)}{\text{no pencils}} = \boxed{7798}$$

(e) How many different groups with exactly 2 wooden pencils (or) exactly 4 pens can she grab?

$$\frac{C(4, 2)}{2 \text{ WP}} \frac{C(12, 4)}{4 \text{ NWP}} + \frac{C(10, 4)}{4 \text{ pens}} \frac{C(6, 2)}{2 \text{ NOT pens}} - \frac{C(4, 2)}{2 \text{ WP}} \frac{C(10, 4)}{4 \text{ pens}}$$

$$= \boxed{4860}$$