Quiz 3

- 5 minute individual quiz;
- Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
- **Show and explain all work;**
- **Underline** the answer of each step;
- The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
- By taking this quiz, you agree to follow the university’s code of academic integrity.

**Exercise 1** 100%

Find the general solution to the following ODE

\[ y'' + y = x \sin(x) \]

and graph the evolution of \( y(x) \) for large values of \( x > 0 \).
Quiz 2: solutions

Exercise 1  100%

We first consider the homogeneous equation by solving the characteristic equation
\[ \lambda^2 + 1 = 0. \]
This is \( \lambda = \pm i \). Therefore two linearly independent solutions of the homogeneous equation are given by
\[ y_1(x) = \text{Re}(e^{ix}) = \cos(x), \quad y_2(x) = \text{Im}(e^{ix}) = \sin(x). \]
We now guess a particular solution of the form
\[ y_p(x) = \text{Im}(z_p(x)), \quad \text{where} \quad z_p(x) = w_p(x)e^{ix}. \]
Plugging \( z_p(x) \) into the ODE we get
\[ w''_p + 2iw'_p = x, \tag{1} \]
leading to the educated guess for \( w_p(x) \)
\[ w_p(x) = Ax^2 + Bx \]
for some constants \( A \) and \( B \). Plugging \( w_p(x) \) in (1) yields
\[ A = -\frac{i}{4}, \quad B = \frac{1}{4}. \]
Therefore,
\[ y_p(x) = \text{Im}(z_p(x)) = \text{Im} \left( \left( -\frac{i}{4}x^2 + \frac{1}{4}x \right)(\cos(x) + i\sin(x)) \right) \]
\[ = \frac{1}{4}x \sin(x) - \frac{x^2}{4} \cos(x). \]
and
\[ y(x) = C_1 \cos(x) + C_2 \sin(x) + \frac{1}{4}x \sin(x) - \frac{x^2}{4} \cos(x), \]
for some constants \( C_1 \) and \( C_2 \).
For large values of \( x \) the solution \( y(x) \) looks like
\[ y(x) \approx -\frac{x^2}{4} \cos(x). \]
The graph is provided in Fig. 1.
Figure 1: graph of $y(x) = -\frac{x^2}{2} \cos(x)$. 