Quiz 4

• 5 minute individual quiz;
• Answer the questions in the space provided. If you run out of space, continue onto the back of the page. Additional space is provided at the end;
• **Show and explain all work;**
• **Underline** the answer of each steps;
• The use of books, personal notes, **calculator**, cellphone, laptop, and communication with others is forbidden;
• By taking this quiz, you agree to follow the university’s code of academic integrity.

Exercise 1 100%

Find the solution of 

\[ y'' + y = g(t), \quad y(0) = y'(0) = 0, \]

where

\[ g(t) := \begin{cases} 
0 & t < 3\pi, \\
2 & t \geq 3\pi. 
\end{cases} \]
Quiz 4: solutions

Exercise 1 100%

We write the right and side using a step function

\[ g(t) = 2u_{3\pi}(t), \]

so that taking the Laplace transform yields

\[ (s^2 + 1)Y = \frac{2}{s}e^{-3\pi s}, \]

where \( Y = \mathcal{L}(y) \). As a consequence, we obtain

\[ Y = \frac{2}{s(s^2 + 1)}e^{-3\pi s}. \]

We first find the Laplace transform inverse of \( F(s) := \frac{2}{s(s^2 + 1)} \) using simple fractions

\[ \frac{2}{s(s^2 + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1} = \frac{2}{s} - \frac{2s}{s^2 + 1}. \]

Hence,

\[ \mathcal{L}^{-1}(F)(t) = 2 - 2\cos(t). \]

Then, using the formula

\[ \mathcal{L}^{-1}(F(s)e^{-ct}) = \mathcal{L}^{-1}(F)(t - c)u_c(t) \]

we arrive at

\[ g(t) = (2 - 2\cos(t - 3\pi))u_{3\pi}(t) = (2 + 2\cos(t))u_{3\pi}(t). \]