Title: From Algebraic to Quantum Complexity

Abstract: A deep problem which remains open is the relation between the complexity classes NP and BQP. Toward understanding this question, we present a natural computational problem that (as an underlying parameter is varied) interpolates between these two famous complexity classes.

More precisely, let UNIFEAS_p denote the problem of deciding whether a univariate polynomial f, with integer coefficients, has a p-adic rational root. Also let UNIFEAS_p(m) denote the analogous problem, restricted to sparse polynomials with m or fewer monomial terms. We show that

(a) UNIFEAS_p(2) in BQP.
(b) UNIFEAS_p in BQP implies that NP is in BQP.

Curiously, the more traditional analogue of UNIFEAS over the real numbers is not even known to be NP-complete. While recent results on Carmichael numbers are used, we assume no background in number theory.