Math 141 Week-in-Review 10
Sections 8.5-8.6, 5.1

Key Terms and Relevant Equations:

- **probability density function**: A function that defines the probability distribution of a random variable $X$. $P(a < X < b)$ is the area between the graph of the probability density function and the x-axis between $x = a$ and $x = b$.

- **normal distribution**: A distribution defined by the mean, $\mu$, and the standard deviation, $\sigma$, with probability density function $p(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$.

- **standard normal distribution**: The normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

- **simple interest**: Interest that is computed on the original payment only.

- **compound interest**: Earned interest that is periodically added to the principal and thereafter itself earns interest at the same rate.

- **effective rate of interest**: The annual rate of interest that, when compounded annually, will yield the same accumulated amount as the nominal rate compounded $m$ times a year (over the same term).

Relevant Equations:

- **simple interest**: $I = Prt$, $A = P(1 + rt)$

- **compound interest**: $A = P \left(1 + \frac{r}{m}\right)^{mt}$

- **continuous compound interest**: $A = Pe^{rt}$

- **effective rate of interest**: $r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$

Probability density functions have the following properties:

1. The domain (possible $x$ values) of the function is the interval of values taken on by the random variable associated with the experiment.

2. The probability density function is always non-negative (either positive or zero).

3. The area of the region between the graph of the function and the x-axis is equal to 1.

4. $P(a < X < b) = \text{area under the probability density function’s curve between the values } x = a \text{ and } x = b$.

5. $P(X = k) = 0$. (Notice that there are infinitely many points to spread the probability over.)

6. $P(X < k) = P(X \leq k)$.

Normal curves have the following properties:

1. The peak of the curve is at $x = \mu$.

2. The curve is symmetric about a vertical line passing through the mean.

3. The curve never crosses the x-axis.

4. The curve approaches but is never equal to 0 on both the positive and negative x-axis.

5. The area under the curve is 1.

6. The standard deviation determines how “steep” or “flat” the curve is (the curve is flatter for large $\sigma$).
1. Let $Z$ be the standard normal variable. Find the following (rounded to four decimal places):

(a) $P(Z < 0.65)$
(b) $P(Z \geq 1.21)$
(c) $P(-0.34 \leq Z \leq 0.88)$

2. Let $X$ be a normal random variable with $\mu = 220$ and $\sigma = 12$. Find the following (rounded to four decimal places):

(a) $P(Z \leq 200)$
(b) $P(Z > 215)$
(c) $P(198 \leq Z \leq 217)$

3. Let $Z$ be the standard normal variable. For each of the following, find the value of $a$ (rounded to four decimal places):

(a) $P(Z \leq a) = 0.6312$
(b) $P(Z \geq a) = 0.2875$
(c) $P(-a \leq Z \leq a) = 0.5876$

4. Let $X$ be a normal random variable with $\mu = -45$ and $\sigma = 8$. For each of the following, find the value of $a$ (rounded to four decimal places):

(a) $P(X \leq a) = 0.4512$
(b) $P(X \geq a) = 0.3143$

5. Let $X$ be a normal random variable with $\mu = -45$ and $\sigma = 8$. Find the values of $a$ and $b$ that are symmetric about the mean such that $P(a < X < b) = 0.5234$ (rounded to four decimal places).

6. The weights in pounds of full-grown Puggles are normally distributed with a mean of 24 and a standard deviation of 3. If a full-grown Puggle is selected at random, what is the probability that it weighs

(a) more than 30 pounds?
(b) less than 20 pounds?
(c) between 21 and 27 pounds?

7. The annual amount of snowfall in inches for Toledo, Ohio is normally distributed with a mean of 37.8 and a standard deviation of 8.

(a) In a given year, what is the probability that it snows more than 50 inches?
(b) How much must it snow in a given year for it to be more than the snowfall of 90% of years.
8. The scores on a math test are normally distributed with a mean of 75 and a standard deviation of 12. If the instructor want to assign A’s to 15%, B’s to 25%, C’s to 30%, D’s to 20%, and F’s to 10% of the class, what should the cutoff points be?

9. The heights of fifth graders at Henderson elementary school are normally distributed with mean 54 inches and standard deviation 2 inches. How tall must a fifth grader be in order to be in the middle 50% of fifth graders at Henderson elementary school?

10. $550 is put into a savings account that earns simple interest at a rate of 4% annually. How much is in the account after 6 months?

11. A savings account earns simple interest at a rate of 6% annually. If the account has $3,968 after 4 years, how much was in the account initially?

12. How much would you have in 10 years if you invest $1,200 in an account with an interest rate of 8% per year compounded monthly?

13. How long will it take for an investment to double if it earns 5% annual interest compounded quarterly?

14. If you invest $3000 at 7% interest per year compounded continuously, how much would you have in your account after 7 months? How much interest is earned during this time?

15. You have an option to put your money into an account that earns 8% interest per year compounded annually or 7.8% interest per year compounded monthly. Which should you choose?