Math 141 Week-in-Review 11
Sections 5.2-5.3 and Final Exam Review

Note: This review is a selection of some problems that are representative of the topics covered in each section, but it is NOT all-inclusive. For the final exam, you should know how to do ALL types of problems found in your class notes, homework, quizzes, and previous exams. For more detailed coverage of each individual section, see the previous Week-in-Reviews.

1. Matthew wants to save up $20,000 over the next five years in order to buy a car. He decides to make quarterly deposits into an account that earns interest at the rate of 8.5%/year compounded quarterly. How much must he deposit each quarter to meet his goal?

\[ N = 20 \]
\[ I\% = 8.5 \]
\[ PV = 0 \]
\[ PMT = 812.94 \]
\[ FV = 20,000 \]
\[ P/Y = C/Y = 4 \]

2. Kevin bought a house for $210,000. He made a down payment of $20,000 and secured a mortgage with interest charged at the rate of 6%/year compounded monthly on the unpaid balance. The loan is to be amortized over 30 years.

(a) What are Kevin’s monthly mortgage payments?

\[ N = 12 \times 30 = 360 \]
\[ I\% = 6 \]
\[ PV = 190,000 \]
\[ PMT = 1139.15 \]
\[ FV = 0 \]
\[ P/Y = C/Y = 12 \]

(b) How much interest does Kevin pay?

\[ \text{Kevin pays: } 1139.15 \times 360 = \$410,094 \]
\[ \text{Interest} = 410,094 - 190,000 = \$220,094 \]
3. Nicole takes out a loan for $45,000 to start a business. In order to pay off the loan in 5 years, she makes monthly payments of $906.

(a) What is her annual interest rate (assuming it is compounded monthly)?

\[
\begin{align*}
N &= 12 \times 5 = 60 & \text{PMT} &= -906 \\
I\% &= 7.7 & \text{FV} &= 0 \\
PV &= 45,000 & P/Y = C/Y &= 12 \\
\end{align*}
\]

(b) What is Nicole’s outstanding principle after 2 years?

\[
\begin{align*}
N &= 24 \times 2 = 48 & \text{PMT} &= -906 \\
I\% &= 7.7 & \text{FV} &= -29,039.99 \\
PV &= 45,000 & P/Y = C/Y &= 12 \\
\end{align*}
\]

\[\text{Outstanding Principle} = 29,039.99\]

(c) If after 2 years, Nicole is given the opportunity to refinance her loan at an interest rate of 5%, what is her new monthly payment? How much does she save by refinancing?

\[
\begin{align*}
N &= 36 \text{ (months remaining)} & \text{PMT} &= -870.35 \\
I\% &= 5 & \text{FV} &= 0 \\
PV &= 29,039.99 & P/Y = C/Y &= 12 \\
\end{align*}
\]

\[\text{New Payment} = 870.35\]

\[\text{Would have paid} = 906 \times 36 = 32,616\]

\[\text{Now} = 870.35 \times 36 = 31,332.60\]

\[\text{Saves} = 32,616 - 31,332.60 = 1283.40\]
Section 1.3: Finding the equation of a line

4. At a unit price of $40, the quantity demanded of a certain video game is 3675 games. When the price is increased to $50, the demand drops to 2625 games.

(a) Given that it is linear, find the demand equation.

\[
\begin{align*}
\text{quant. demanded} & \quad \text{price} \\
3675 & \quad 40 \\
2625 & \quad 50
\end{align*}
\]

\[
m = \frac{\Delta p}{\Delta x} = \frac{50 - 40}{2625 - 3675} = \frac{10}{-1050} = -\frac{1}{105}
\]

\[
p = mx + b
\]

\[
40 = -\frac{1}{105}(3675) + b \\
40 = -35 + b \\
b = 75
\]

\[
p = -\frac{1}{105}x + 75
\]

(b) Above what price will there be no demand?

\[
x = 0 \\
p = 75
\]

Section 1.4: Finding the intersection of 2 lines

5. A certain company that manufactures vacuum cleaners has a monthly fixed cost of $6000. When they make and sell 80 vacuums, the total cost is $9200 and the revenue is $7200. Assuming that the cost and revenue functions are linear, how many vacuums must they produce and sell to break even?

\[
C(x) = cx + F \\
C(x) = cx + 6000 \\
9200 = c \cdot 80 + 6000 \\
3200 = 80c \\
40 = c \\
\]

\[
R(x) = sx \\
7200 = s \cdot 80 \\
s = 90 \\
R(x) = 90x
\]

\[
y_1 = 40x + 6000 \\
y_2 = 90x \\
S: \text{intersect} \\
(120, 10800)
\]

120 vacuums
Section 1.Q: Quadratic equations (finding vertices and/or x-intercepts)

6. The cost to produce $x$ pairs of a certain brand of shoe is given by the function $C(x) = 40x + 12,000$. The relationship between the price, $p$, and the demand, $x$, is given by the equation $p(x) = -\frac{1}{40}x + 120$. How many pairs of shoes must be sold to maximize the profit?

\[
R(x) = x \cdot p(x) = -\frac{1}{40}x^2 + 120x
\]

\[
P(x) = R(x) - C(x) = -\frac{1}{40}x^2 + 120x - (40x + 12000)
\]

\[
= -\frac{1}{40}x^2 + 80x - 12000
\]

$a < 0,$

\[
\max = \text{vertex} = -\frac{b}{2a} = -\frac{80}{2(-\frac{1}{40})} = 1600 \text{ pairs of shoes}
\]

Section 2.1: Systems of Linear Equations (solve without matrices, write system from word problem)

7. 13,000 tickets were sold for a recent Texas A&M basketball game. Mezzanine tickets cost $20, balcony tickets cost $10, and student tickets cost $5. The total revenue for ticket sales was $170,000. The number of mezzanine tickets was twice the number of balcony tickets. How many of each type of ticket were sold? Set up the system of equations used to solve this problem, but do not solve.

\[
m = \# \text{ of mezzanine tickets sold}
\]

\[
b = \# \text{ of balcony tickets sold}
\]

\[
s = \# \text{ of student tickets sold}
\]

\[
m + b + s = 13,000
\]

\[
20m + 10b + 5s = 170,000
\]

\[
m = 2b \rightarrow m - 2b = 0
\]
Section 2.2: Gauss-Jordan Elimination (unique solution)

8. Pivot the given matrix about the circled element.

\[
\begin{bmatrix}
1 & 4 & 2 & 6 \\
0 & 3 & -6 & 12 \\
0 & 1 & -3 & 3
\end{bmatrix}
\xrightarrow{\frac{1}{3}R_2}
\begin{bmatrix}
1 & 4 & 2 & 6 \\
0 & 1 & -2 & 4 \\
0 & 1 & -3 & 3
\end{bmatrix}
\xrightarrow{R_1 - 4R_2}
\begin{bmatrix}
1 & 0 & 10 & -10 \\
0 & 1 & -2 & 4 \\
0 & 0 & -1 & -1
\end{bmatrix}
\]

Section 2.3: Gauss-Jordan Elimination (no solutions or infinitely many solutions)

9. Solve the following system of linear equations

\[
\begin{align*}
2x + 3y + 13z &= 23 \\
-x + 2y + 4z &= 6 \\
x + y + 5z &= 9
\end{align*}
\]

\[
\begin{bmatrix}
x & y & z & \text{const} \\
2 & 3 & 13 & 23 \\
-1 & 2 & 4 & 6 \\
1 & 1 & 5 & 9
\end{bmatrix}
\xrightarrow{\text{rref}}
\begin{bmatrix}
1 & 0 & 2 & 4 \\
0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
x + 2z = 4 \\
y + 3z = 5
\]

\[
\begin{aligned}
z &= t \\
x + 2t &= 4 \\
x &= 4 - 2t \\
y &= 5 - 3t
\end{aligned}
\]

\[
(x, y, z) = (4 - 2t, 5 - 3t, t)
\]
Section 2.4: Matrices (Addition, Transpose, Scalar Multiplication)

10. Find the value of $x$, $y$, and $z$ that make the following matrix equation true.

$$3 \begin{bmatrix} -1 & 3 \\ 2 & x \end{bmatrix} + \begin{bmatrix} 2 & y \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}^T$$

$$\begin{bmatrix} -3 & 9 \\ 6 & 3x \end{bmatrix} + \begin{bmatrix} 2 & y \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ z \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 9+y \\ 3 & 3x+5 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ z \\ -7 \end{bmatrix}$$

$$x = -1$$  

$q+y = 4$  

$z = 3$  

$3x+5 = -7$  

$3x = -12$  

$x = -4$

$y = -5$

Section 2.5: Multiplication of Matrices

11. Find the value of $x$ and $y$ that make the following matrix equation true.

$$\begin{bmatrix} x & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -x-2y & 2x-6 \\ -2-y & 4-3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-x-2y = 1$  

$2x-6 = 0$  

$-2-y = 0$  

$1 = 1$

$2x = 6$  

$y = -2$

$x = 3$
Section 3.1: Graphing systems of inequalities

12. Determine graphically the solution set for the following system of inequalities. Indicate whether it is bounded or unbounded.

\[
\begin{align*}
4x + y & \geq 16 \\
5x + 4y & \geq 40 \\
x + 5y & \geq 20 \\
x & \geq 0, y & \geq 0
\end{align*}
\]

Section 3.2: Setting up linear programming problems

13. A nutritionist needs to prepare a diet for one of her patients. She has decided that the meals will consist of Food A and Food B and that they should contain 30 mg of Vitamin C, 360 mg of Calcium, and 15 mg of Iron. Each ounce of Food A contains 2.5 mg of Vitamin C, 15 mg of Calcium, 0.5 mg of Iron, and 3 mg of cholesterol. Each ounce of Food B contains 1 mg of Vitamin C, 15 mg of Calcium, 0.75 mg of Iron, and 4 mg of cholesterol. How many ounces of each type of food should be used in each meal so that the cholesterol content is minimized and the minimum requirements of Calcium, Iron, and Vitamin C are met? Set up the linear programming problem, but do NOT solve.

\[
\begin{align*}
x &= \text{# of oz. of Food A} \\
y &= \text{# of oz. of Food B}
\end{align*}
\]

Minimize \( C = 3x + 4y \)

Subject to \( 2.5x + y \geq 30 \) - Vit C
\( 15x + 15y \geq 360 \) - Calc.
\( 0.5x + 0.75y \geq 15 \) - Iron
\( 7x \geq 0, y \geq 0 \).
Section 3.3: Solving linear programming problems (method of corners)

14. Solve the linear programming problem that was set up in the previous problem. What is the minimum cholesterol content?

\[
egin{align*}
2.5x + y &\geq 30 \\
y &\geq -2.5x + 30 \\
15y &\geq -15x + 360 \\
y &\geq -x + 24 \\
0.5x + 0.75y &\geq 15 \\
0.75y &\geq -0.5x + 15 \\
y &\geq -\frac{2}{3}x + 20
\end{align*}
\]

\[
\begin{array}{|c|c|}
\hline
\text{CP} & \text{C = 3x + 4y} \\
\hline
(0,30) & 120 \\
(4,30) & 92 \\
(12,12) & 84 \\
(30,0) & 90 \\
\hline
\end{array}
\]

12 oz. of Food A and 12 oz. of Food B for a minimum cholesterol content of 84 mg

Section 6.1: Sets and set operations

15. Shade the region \( (A \cap C) \cup (B \cap C^c) \) in the Venn Diagram.
Section 6.2: Number of elements in a set

16. A survey of 200 movie watchers was conducted to determine which genres of movies they like. 

Let C be the set of people who like comedy. 
Let A be the set of people who like action. 
Let S be the set of people who like science fiction.

Use the survey results below to find the number of people in each section of the given Venn diagram.

- 130 people like comedy. 
- 118 people like action. 
- 105 people like science fiction. 
- 68 people like comedy and science fiction. 
- 75 people like action and science fiction. 
- 23 people like action and science fiction but not comedy. 
- 21 people only like comedy.

Find the number of people who like comedy or action but not science fiction.

\[a + b + c = 21 + 41 + 2 = 64\]

Section 6.3: Multiplication principle

17. How many different license plates can be made using three letters followed by three digits (0-9) if the first letter is a C and no letter or number can be repeated?

\[1 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 432,000\]
Section 6.4: Permutations and Combinations

18. A jar of marbles contains 7 yellow marbles, 5 blue marbles, and 6 red marbles. How many ways can a sample of 6 marbles be selected so that exactly 2 are yellow or exactly 3 are red?

\[ n(2Y \cup 3R) = n(2Y) + n(3R) - n(2Y \cap 3R)\]

\[ n(2Y) = \binom{7}{2} \cdot \binom{11}{4} = 6,930 \]

\[ n(3R) = \binom{6}{3} \cdot \binom{12}{3} = 4,400 \]

\[ n(2Y \cap 3R) = \binom{7}{2} \cdot \binom{6}{3} \cdot \binom{5}{1} = 2,100 \]

\[ n(2Y \cup 3R) = 6,930 + 4,400 - 2,100 = 9,230 \]

Section 7.1: Language of Probability

19. Consider the experiment of rolling a fair six-sided die and observing the uppermost number. How many events does this experiment have?

\[ S = \{1, 2, 3, 4, 5, 6\} \]

\[ \# \text{ events} = 2^6 = 64 \]

Section 7.2: Definition of Probability

20. Let \( E_1, E_2, E_3, E_4, E_5, E_6 \) be six mutually exclusive events that together compose the entire sample space. Consider the following probability distribution

<table>
<thead>
<tr>
<th>Event</th>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
<th>( E_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.15</td>
<td>0.25</td>
<td>0.2</td>
<td>0.05</td>
<td>0.1</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Find \( P(E_2 \cup E_5) \).

\[ P(E_2 \cup E_5) = P(E_2) + P(E_5) = 0.25 + 0.1 = 0.35 \]

Section 7.3: Rules of Probability

21. An experiment consists of drawing a card from a standard 52-card deck. What is the probability that the card is a red card or a face card (jack, queen, king)?

\[ P(\text{Red } \cup \text{ Face}) = P(\text{Red}) + P(\text{Face}) - P(\text{Red } \cap \text{ Face}) \]

\[ = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13} \]
Section 7.4: Use of Counting Techniques in Probability

22. Four cards are drawn without replacement from a standard 52-card deck. What is the probability that exactly two of them are diamonds?

\[
\text{Total # of ways to draw 4 cards: } C(52,4) = 270,725
\]
\[
\text{# of ways to get 2 diamonds: } C(13,2) \cdot C(39,2) = 57,798
\]
\[
\frac{57,798}{270,725} \approx 0.2135
\]

Section 7.5: Conditional Probability and Independent Events

23. Let \( E \) and \( F \) be two events such that \( P(E) = 0.2, P(F) = 0.8 \) and \( P(E \cup F) = 0.84 \).

Are \( E \) and \( F \) independent?

Check \( P(E \cap F) = P(E) \cdot P(F) \) ?.

\[
P(E \cup F) = P(E) + P(F) - P(E \cap F)
\]
\[
0.84 = 0.2 + 0.8 - P(E \cap F)
\]
\[
P(E \cap F) = 0.16
\]

\[
P(E) \cdot P(F) = 0.2 \cdot 0.8 = 0.16
\]

[Yes]

Section 7.6: Bayes Theorem

24. There are three jars that each contain 10 marbles. The first contains 2 white marbles and 8 red marbles, the second 7 white and 3 red, and the third 5 white and 5 red. An experiment consists of first selecting a jar at random. (Assume each jar has an equal probability of being selected.) After a jar is selected, a marble is randomly drawn from this jar, noting its color. If the marble drawn was white, find the probability that the second jar was selected.

\[
P(2 | w) = \frac{P(2 \cap w)}{P(w)}
\]
\[
= \frac{1/3 \cdot 7/10}{1/3 \cdot 2/10 + 1/3 \cdot 7/10 + 1/3 \cdot 5/10}
\]
\[
= \frac{7/30}{14/30}
\]
\[
= \frac{1}{2}
\]
Section 8.1: Distributions of Random Variables

25. Draw in the missing bar in the Venn Diagram and find \( P(3 \leq X < 5) \).

\[
P(3 \leq X < 5) = P(X = 3) + P(X = 4)
= 0.1 + 0.2
= 0.3
\]

Section 8.2: Expected Value and Measures of Central Tendency

26. In a lottery, 5000 tickets are sold. One first prize of $6000, one second prize of $2000, three third prizes of $500 and ten consolation prizes of $50 are to be awarded. What is the price of a ticket if this is a fair game?

\[
E(x) = 0.
\]

\[
\begin{array}{c|c|c|c|c|c}
X & 6000-a & 2000-a & 500-a & 50-a & -a \\
P(x) & \frac{1}{5000} & \frac{1}{5000} & \frac{3}{5000} & \frac{10}{5000} & \frac{4985}{5000} \\
\end{array}
\]

\[
E(x) = (6000-a)\left(\frac{1}{5000}\right) + (2000-a)\left(\frac{1}{5000}\right) + (500-a)\left(\frac{3}{5000}\right) + (50-a)\left(\frac{10}{5000}\right) - a\left(\frac{4985}{5000}\right)
= 2 - a
\]

\[
a = 2
\]

Section 8.3: Variance and Standard Deviation

27. Find the variance and standard deviation of the random variable \( X \) having the following probability distribution. (Round to 4 decimal places)

\[
\begin{array}{c|c|c|c|c|c}
X & 0 & 5 & 10 & 15 & 20 \\
\hline
\text{Probability} & \frac{1}{3} & \frac{1}{6} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} \\
\end{array}
\]

\[
\text{St. dev.} = \sigma x = \sqrt{7.2048} \\
\text{Var} (x) = \sigma^2 x = 51.9097
\]
Section 8.4: Binomial Distribution

28. An exam consists of 20 multiple choice questions, each with four choices. If a student randomly guesses on each question, what is the probability that they answer more than 4 and at most 12 questions correctly?

\[ n = 20 \quad p = \frac{1}{4} \]

\[ 6, 7, \ldots, 11, 12 \]

\[ \text{binomcdf}(20, \frac{1}{4}, 12) - \text{binomcdf}(20, \frac{1}{4}, 4) = 0.5850 \]

Section 8.5: Normal Distribution

29. Let \( Z \) be the standard normal variable. Find the value of \( a \) such that \( P(-a < Z < a) = 0.8164 \).

\[ \mu = 0 \quad \sigma = 1 \]

\[ 1 - 0.8164 = 0.1836 \]

\[ \frac{0.1836}{2} = 0.0918 \]

\[ \alpha = \text{invNorm}(0.0918 + 0.8164, 0, 1) = 1.3298 \]

Section 8.6: Applications of Normal Distributions

30. The birth weights of hippos are normally distributed with mean 80 lbs. and standard deviation 10 lbs. If a newborn hippo is chosen at random, what is the probability that it weighs more than 100 lbs?

\[ \mu = 80 \quad \sigma = 10 \]

\[ P(X > 100) = \text{normalcdf}(100, 1\infty, 80, 10) = 0.0228 \]

Section 5.1: Interest

31. A savings account has an interest rate of 4% per year compounded continuously. If there is $2000 in the account initially, how much is in the account after 18 months?

\[ A = P e^{rt} \]

\[ = 2000 e^{0.04 \cdot \frac{18}{12}} = 2123.67 \]