Math 141 Week-in-Review 2
Sections 1.Q,2.1-2.2

Relevant Equations:

Standard Form: \( y = ax^2 + bx + c \)

Vertex Form: \( y = a(x - h^2) + k \)

Vertex: \( h = -\frac{b}{2a} \)

Quadratic Formula: \( x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}, \quad \text{if} \quad b^2 - 4ac \geq 0 \)

Revenue: \( R(x) = xp(x) \)

Key Terms:

- **parabola**: the graph of a quadratic function
- **vertex**: the point on the graph of a parabola that has either the highest (when opening down) or the lowest (when opening up) y-value
- **real zeros (roots)**: the x-intercepts of the function
- **dependent system**: a system of equations with infinitely many solutions
- **inconsistent system**: a system of equations with no solutions
- **Gauss-Jordan Elimination**: the process of transforming an matrix to Row-Reduced form

A matrix is in Row-Reduced Form when:

1. Each row consisting entirely of zeros lies below all rows having nonzero entries.
2. The first nonzero entry in each (nonzero) row is 1 (called a **leading 1**).
3. In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
4. If a column in the coefficient matrix contains a leading 1, then the other entries in that column are zeros.

1. Are each of the following in Row-Reduced form? If not, give the rule that it breaks.

   \[
   (a) \begin{bmatrix}
   1 & 0 & 2 & 0 \\
   0 & 1 & 0 & 2 \\
   0 & 0 & 0 & 1 
   \end{bmatrix} \quad (b) \begin{bmatrix}
   0 & 1 & 0 & 2 \\
   1 & 0 & 0 & 5 \\
   0 & 0 & 1 & 1 
   \end{bmatrix} \quad (c) = \begin{bmatrix}
   1 & 0 & 2 & 3 \\
   0 & 1 & 0 & -3 \\
   0 & 0 & 1 & 2 
   \end{bmatrix} \\
   (d) \begin{bmatrix}
   1 & 0 & 0 & 4 \\
   0 & 1 & 0 & 4 \\
   0 & 0 & 3 & 5 
   \end{bmatrix} \quad (e) = \begin{bmatrix}
   1 & 0 & 2 \\
   0 & 1 & 3 \\
   0 & 0 & 0 
   \end{bmatrix} \quad (f) = \begin{bmatrix}
   1 & 0 & 0 & 2 \\
   0 & 0 & 0 & 0 \\
   0 & 0 & 1 & 3 
   \end{bmatrix}
   \]
2. Solve the following system of equations using Gauss-Jordan elimination.

\[
\begin{align*}
2x + 4y - 6z &= 38 \\
x + 2y + 3z &= 7 \\
3x - 4y + 4z &= -19
\end{align*}
\]

3. A guitar manufacturer found that when the price of a certain model is $500, 4000 guitars are sold each month. For every increase in price of $20, the demand decreases by 300. The manufacturer has a fixed cost of $300,000 per month and it costs $200 to make each guitar.

a) Find the profit equation.
b) What is the maximum profit (rounded to the nearest cent)? How many guitars should be sold to achieve this maximum?
c) Approximately how many guitars should be sold for the company to break even?

4. Find all solutions to the following systems of equations, whenever they exist. Use any method.

(a) \[
\begin{align*}
2x - 3y &= 5 \\
-4x + 6y &= -10
\end{align*}
\]

(b) \[
\begin{align*}
x - 2y &= -3 \\
2x + 3y &= 8
\end{align*}
\]

(c) \[
\begin{align*}
-3x + 9y &= 2 \\
4x - 12y &= 8
\end{align*}
\]

5. Pivot about the entry in the second row and second column.

\[
\begin{bmatrix}
1 & 2 & -4 & -3 & 0 \\
0 & 2 & 10 & 6 & 2 \\
0 & -12 & 22 & -2 & 4 \\
0 & 3 & 5 & 5 & 9
\end{bmatrix}
\]

6. Consider the following augmented matrix.

\[
\begin{bmatrix}
1 & 3 & -4 \\
-2 & a & 8
\end{bmatrix}
\]

For what value of a will the system be dependent?

7. A parabola has a vertex at (1,2) and also goes through the point (4,-25). Find the equation of the quadratic function. Is the y-coordinate of the vertex a maximum or a minimum of the function?
8. A high school football game charges $5 for tickets for children, $7 for students, and $10 for adults. At a certain game, 900 people attended and the total ticket sales were $6690. The number of student tickets sold was twice the number of child tickets sold. How many of each type of ticket was sold?

9. (Adapted from Tan #38 2.3) A dietitian wishes to plan a meal around three foods. The meal is to include 8800 units of vitamin A, 3380 units of vitamin C and 1040 units of calcium. The number of units of the vitamins and calcium in each ounce of the foods is summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Food I</th>
<th>Food II</th>
<th>Food III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitamin A</td>
<td>400</td>
<td>1200</td>
<td>800</td>
</tr>
<tr>
<td>Vitamin C</td>
<td>110</td>
<td>570</td>
<td>340</td>
</tr>
<tr>
<td>Calcium</td>
<td>90</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

Determine the amount of each food the dietitian should include in the meal in order to meet the vitamin and calcium requirements.