Math 141 Week-in-Review 3
Sections 2.3-2.5 and Review of Ch. 1-2

Key Terms:

- **matrix**: an ordered rectangular array of numbers. A matrix with \(m\) rows and \(n\) columns has size \(m \times n\). The entry in the \(i\)th row and \(j\)th column of a matrix \(A\) is denoted by \(a_{ij}\).
- **row matrix (or row vector)**: a \(1 \times n\) matrix.
- **column matrix (or column vector)**: an \(n \times 1\) matrix.
- **square matrix**: an \(n \times n\) matrix.
- **matrix equality**: two matrices are equal if they have the same size and their corresponding entries are the same.
- **identity matrix**: a square matrix with 1’s on the main diagonal and 0’s everywhere else.

Matrix Operations:

- **Addition and Subtraction**: Two matrices of the same size can be added (or subtracted) to obtain another matrix of the same size by adding (or subtracting) corresponding entries.
- **Transpose**: If \(A\) is an \(m \times n\) matrix with elements \(a_{ij}\) then the **transpose** of \(A\) is the \(n \times m\) matrix \(A^T\) with elements \(a_{ji}\). In other words, the rows of \(A\) are the columns of \(A^T\), and the columns of \(A\) are the rows of \(A^T\).
- **Scalar Multiplication**: If \(c\) is a number and \(A\) is a matrix, then the matrix \(cA\) is the matrix obtained by multiplying every entry in \(A\) by \(c\).
- **Matrix Multiplication**: If \(C = AB\), then the entry \(c_{ij}\) is the result of multiplying the \(i\)th row of \(A\) by the \(j\)th row of \(B\).

1. Solve the following system of equations.

   \[
   \begin{align*}
   x - 2y + z &= -3 \\
   2x + y - 2z &= 2 \\
   x + 3y - 3z &= 6
   \end{align*}
   \]
2. Solve the following system of equations.

\[
\begin{align*}
    x - 3y + 2z &= 7 \\
    2x + 2y - 4z &= -2 \\
    3x - y - 2z &= 5
\end{align*}
\]

3. Solve the following system of equations.

\[
\begin{align*}
    x + 2y &= 4 \\
    x - 2y &= 0 \\
    4x + 3y &= 12
\end{align*}
\]
4. Solve the following system of equations.

\[ w + 2x - 3y + z = -2 \]
\[ 3w - x - 2y - 4z = 1 \]
\[ 2w + 3x - 5y + z = -3 \]
5. Let $A$, $B$, and $C$ be the following matrices:

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -3 & 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -2 & 6 \\ 1 & -5 & -4 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & -2 \\ 3 & -1 \\ 4 & 6 \end{bmatrix}$$

a) Compute $2A - 3B$

b) Compute $3A + C^T$

6. Find the values of $x$, $y$ and $z$ that satisfy the following matrix equation.

$$3 \begin{bmatrix} 2 & 1 \\ x & -4 \end{bmatrix} + \begin{bmatrix} y & 3 \\ -5 & 7 \end{bmatrix}^T = \begin{bmatrix} -2 & z \\ 3 & -5 \end{bmatrix}$$
7. Let $A$ be a $2 \times 5$ matrix, $B$ a $5 \times 3$ matrix, and $C$ a $2 \times 3$ matrix. What are the sizes of the given matrices?

a) $M = AB + C$

b) $M = C^T A + B$

c) $M = B^T A^T + C^T$

8. Perform the following matrix multiplication

$$
\begin{bmatrix}
3 & 2 \\
1 & -2
\end{bmatrix}
\begin{bmatrix}
6 & 2 \\
4 & 1
\end{bmatrix}
$$
9. Find the matrix $B$ such that $AB = I_2$ where $I_2$ is the $2 \times 2$ identity matrix and

$$A = \begin{bmatrix} 3 & -5 \\ 1 & -2 \end{bmatrix}$$
Section 1.3: Finding the equation of a line

10. A phone manufacturer has a monthly fixed cost of $125,000. When they produce 2300 phones, the total cost is $194,000. Find the cost function, assuming it is linear.

Section 1.4: Finding the intersection of 2 lines

11. The quantity demanded of a certain brand of television is 2100 per week when the unit price is $400. For each increase in price of $10, the demand decreases by 140. The suppliers will not market the televisions if the price is $350 or lower. At a unit price of $500, they are willing to market 3900 televisions. Assuming that the supply and demand equations are linear, find the equilibrium quantity and price.
Section 1.Q: Quadratic equations (finding vertices and/or x-intercepts)

12. Market research has shown that the monthly demand for a certain camera is given by the equation \( p(x) = -\frac{1}{20}x + 380 \). The company that produces the cameras has a monthly fixed cost of $312,000. It costs $60 to produce each camera.

a) Find the revenue and cost functions.

b) Find the profit function.

c) How many cameras should be sold to maximize profit? What is the maximum profit?

d) How many cameras should be sold to break even?
Section 2.1: Systems of Linear Equations (solve without matrices, write system from word problem)

13. Freebirds sells 3 different sizes of burrito: Hybrid, Freebird, and Monster. The Hybrid costs $6, the Freebird costs $7 and the Monster costs $8. On a certain day, a particular location of Freebirds sold $10,920 in burritos. They sold 1620 burritos in total, and they sold three times as many Hybrid burritos as they sold Monster burritos. How many of each type of burrito did they sell? **Set up the system of equations to solve the problem, but do not solve.**

Section 2.2: Gauss-Jordan Elimination (unique solution)

14. Pivot the augmented matrix about the circled element.

\[
\begin{bmatrix}
1 & 2 & 1 & 4 \\
0 & \textcircled{3} & -6 & 9 \\
0 & -4 & 1 & 2
\end{bmatrix}
\]