Math 141 Week-in-Review 4
Sections 3.1-3.3

Key Terms:

- **bounded**: A solution set of a system of linear inequalities is **bounded** if it can be enclosed by a circle.
- **unbounded**: A solution set of a system of linear inequalities is **unbounded** if it CANNOT be enclosed by a circle.
- **Linear Programming Problem**: A problem consisting of a linear objective function to be maximized or minimized subject to certain constraints in the form of linear equations or inequalities.
- **Feasible Region (or feasible set)**: The set of points that satisfy all of the constraints in a linear programming problem.

Rules for graphing linear inequalities:

1. To graph the inequality \( y < mx + b \), graph \( y = mx + b \) as a dashed line and shade below.
2. To graph the inequality \( y \leq mx + b \), graph \( y = mx + b \) as a solid line and shade below.
3. To graph the inequality \( y > mx + b \), graph \( y = mx + b \) as a dashed line and shade above.
4. To graph the inequality \( y \geq mx + b \), graph \( y = mx + b \) as a solid line and shade above.

Method of Corners:

1. Graph the corresponding equations by hand or on your calculator (i.e. solve each inequality for \( y \) and replace the inequality sign with an equal sign). Be careful in choosing your window. If there are non-negativity constraints on \( x \) and \( y \), account for these by choosing \( x\)-min and \( y\)-min to be 0.
2. Determine the feasible region.
3. Find the coordinates of the corner points. You can either do this algebraically or on the calculator. There are four possible corner points:
   - The origin (i.e. the point \((0, 0)\))
   - The intersection of two lines (use 2nd TRACE 5:intersect)
   - The intersection of a line and the y-axis (recognize that this is the y-intercept of the line or use 2nd TRACE 1:value and enter in 0 for \( x \))
   - The intersection of a line and the x-axis (plug in 0 for \( y \) and solve for \( x \) or use 2nd TRACE 2:zero and specify a left and right bound that includes the point where the line actually crosses the x-axis)
4. Make a table and evaluate the objective function at each corner point.
1. Find the graphical solution of the inequality $2x - 3y \leq 12$.

2. Determine graphically the solution set for the following system of inequalities. Indicate whether it is bounded or unbounded.

   \[
   \begin{align*}
   2x - y &\geq -1 \\
   x + y &\leq 8 \\
   3x + 2y &\geq 5 \\
   x &\geq 0, y \geq 0
   \end{align*}
   \]
3. Determine graphically the solution set for the following system of inequalities. Indicate whether it is bounded or unbounded.

\[2x + 3y \geq 12\]
\[2x + y \geq 6\]
\[x \geq 0, y \geq 0\]

4. Solve the following linear programming problem

Maximize \( P = 3x + 2y \)
Subject to \( x - y \geq -1 \)
\( 5x + 3y \leq 35 \)
\( x \geq 0, y \geq 0 \)
5. Solve the following linear programming problem

Minimize \( C = 90x + 40y \)
Subject to
\[
\begin{align*}
2x - y & \geq -2 \\
x - 2y & \leq 6 \\
6x + 7y & \leq 74 \\
3x + y & \geq 12 \\
x & \geq 0, y \geq 0
\end{align*}
\]
6. (Tan 3.3 #30) National Business Machines manufactures two models of portable printers: A and B. Each model A costs $100 to make, and each model B costs $150. The profits are $30 for each model A and $40 for each model B portable printer. If the total number of portable printers demanded per month does not exceed 2500 and the company has earmarked no more than $600,000/month for manufacturing costs, how many units of each model should National make each month to maximize its monthly profit? What is the optimal profit?
7. (Tan 3.3 #36) A financier plans to invest up to $500,000 in two projects. Project A yields a return of 10% on the investment, whereas Project B yields a return of 15% on the investment. Because the investment in Project B is riskier than the investment in Project A, the financier has decided that the investment in Project B should not exceed 40% of the total investment. How much should she invest in each project to maximize the return on her investment? What is the maximum return?
8. (Tan 3.3 #43) The water-supply manager for a Midwestern city needs to supply the city with at least 10 million gallons of potable (drinkable) water per day. The supply may be drawn from the local reservoir or from a pipeline to an adjacent town. The local reservoir has a maximum daily yield of 5 million gallons of potable water, and the pipeline has a maximum daily yield of 10 million gallons. By contract, the pipeline is required to supply a minimum of 6 million gallons/day. If the cost for 1 million gallons of reservoir water is $300 and that for pipeline water is $500, how much water should the manager get from each source to minimize the daily water costs for the city? What is the minimum daily cost?
9. (Tan 3.3 #53) Trendex, a telephone survey company, has been hired to conduct a television-viewing poll among urban and suburban families in the Los Angeles area. The client has stipulated that a maximum of 1500 families is to be interviewed. At least 500 urban families must be interviewed, and at least half of the total number of families interviewed must be from the suburban area. For this service, Trendex will be paid $6000 plus $8 for each completed interview. From previous experience, Trendex has determined that it will incur an expense of $4.40 for each successful interview with an urban family and $5 for each successful interview with a suburban family. How many urban and suburban families should Trendex interview to maximize its profit? What is the optimal profit?