Math 141 Week-in-Review 5
Sections 6.1-6.3

Key Terms:
- **set**: a well-defined collection of objects
- **element**: an object in a set
- **Roster Notation**: the elements of a set are written out
- **Set-builder notation**: the set is defined in terms of its properties
- **subset**: a set $A$ is a subset of $B$ if every element of $A$ is also an element of $B$
- **proper subset**: $A$ is a proper subset of $B$ if $A$ is a subset of $B$ and $A \neq B$
- **empty set**: the set with no elements
- **universal set**: the set of all elements of interest in a particular matter

Set notation:
1. $x \in A$ means $x$ is an element of $A$
2. $A \subseteq B$ means $A$ is a subset of $B$
3. $A \subset B$ means $A$ is a proper subset of $B$
4. $\emptyset$ or $\{\}$ denotes the empty set
5. $U$ denotes the universal set
6. $n(A)$ denotes the number of elements in the set $A$

Complement Rules:  

<table>
<thead>
<tr>
<th>Complement Rules</th>
<th>Laws for Set Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $U^C = \emptyset$</td>
<td>(1) $A \cup B = B \cup A$</td>
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<tr>
<td>(2) $\emptyset^C = U$</td>
<td>(2) $A \cap B = B \cap A$</td>
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<tr>
<td>(3) $(A^C)^C = A$</td>
<td>(3) $A \cup (B \cup C) = (A \cup B) \cup C$</td>
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<tr>
<td>(4) $A \cup A^C = U$</td>
<td>(4) $A \cap (B \cap C) = (A \cap B) \cap C$</td>
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<tr>
<td>(5) $A \cap A^C = \emptyset$</td>
<td>(5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$</td>
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</tbody>
</table>

Counting Rules: 

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<tbody>
<tr>
<td>(1) $n(\emptyset) = 0$</td>
<td>(1) $A \cup B = B \cup A$</td>
</tr>
<tr>
<td>(2) If $A$ and $B$ are disjoint, then $n(A \cup B) = n(A) + n(B)$.</td>
<td>(2) $A \cap B = B \cap A$</td>
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<tr>
<td>(3) $n(A) + n(A^C) = n(U)$</td>
<td>(3) $A \cup (B \cup C) = (A \cup B) \cup C$</td>
</tr>
<tr>
<td>(4) For any sets $A$ and $B$, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.</td>
<td>(4) $A \cap (B \cap C) = (A \cap B) \cap C$</td>
</tr>
<tr>
<td>(5) For any sets $A$, $B$, and $C$, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.</td>
<td>(5) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$</td>
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Multiplication Principle: Suppose there are $m$ ways of performing a task $T_1$ and $n$ ways of performing a task $T_2$. Then there are $mn$ ways of performing the task $T_1$ followed by the task $T_2$.

Generalized Multiplication Principle: Suppose a task $T_1$ can be performed in $N_1$ ways, a task $T_2$ can be performed in $N_2$ ways, ..., and finally, a task $T_m$ can be performed in $N_m$ ways. Then the number of ways of performing the tasks $T_1$, $T_2$, ..., $T_m$ in succession is given by the product $N_1N_2\cdots N_m$.
1. Let \( A = \{1, 2, 3\} \), \( B = \{2, 3, 4\} \), and \( C = \{2, 4\} \).

a) TRUE or FALSE: \( 3 \in B \)

b) TRUE or FALSE: \( \{2\} \in C \)

c) TRUE or FALSE: \( \{2\} \subseteq C \)

d) TRUE or FALSE: \( C \subset B \)

e) TRUE or FALSE: \( C \subseteq A \)

f) TRUE or FALSE: \( 3 \subseteq A \)

g) TRUE or FALSE: \( A \cap C \subseteq B \)

\[ A \cap C = \{2\} \]

2. List all the subsets of the set \( A = \{x | x \text{ is an integer between 4 and 6, inclusive}\} \). How many proper subsets are there?

\[ A = \{4, 5, 6\} \]

\[ \emptyset \]
\[ \{4\} \]
\[ \{5\} \]
\[ \{6\} \]
\[ \{4, 5\} \]
\[ \{4, 6\} \]
\[ \{5, 6\} \]
\[ \{4, 5, 6\} \]

7 proper subsets

3. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), \( A = \{x \in U | x \text{ is even}\} \), \( B = \{3, 4, 6, 8, 9\} \), and \( C = \{3, 5, 9\} \). Find the following sets.

\( A = \{2, 4, 6, 8, 10\} \)

a) \( B^C = \{1, 2, 5, 7, 10\} \)

b) \( A \cap B = \{4, 6, 8\} \)

c) \( A \cup C = \{2, 3, 4, 5, 6, 8, 9, 10\} \)

d) \( (B \cup C)^C = \{1, 2, 7, 10\} \)

\( B \cup C = \{3, 4, 5, 6, 8, 9\} \)

e) \( A^C \cap (B \cup C) = \{3, 5, 9\} \)

\( A^C = \{1, 3, 5, 7, 9\} \)

f) \( (B \cap A) \cup (C \cap A^C) = \{3, 4, 5, 6, 8, 9\} \)

\( B \cap A = \{4, 6, 8\} \)

\( C \cap A^C = \{3, 5, 9\} \)

g) \( (B \cap C)^C \cap A = \{2, 4, 6, 8, 10\} \)

\( B \cap C = \{3, 9\} \)

\( (B \cap C)^C = \{1, 2, 4, 5, 6, 7, 8, 10\} \)
4. Shade the appropriate region in the Venn Diagram.

\[ B \cap A^C \]

\[ (A \cup B) \cap C^C \]

5. Shade the appropriate region in the Venn Diagram.

\[ (A \cap B^C) \cup (B \cap C) \]

\[ A \cap B^C \]

\[ B \cap C \]

\[ (A \cap B^C) \cup (B \cap C) = \{a, d, e, f\} \]

\[ U = \{a, b, c, d, e, f, g, h\} \]

\[ A = \{a, b, d, e\} \]

\[ B = \{b, c, e, f\} \quad B^c = \{a, d, g, h\} \]

\[ C = \{d, e, f, g\} \]

\[ A \cap B^c = \{a, d\} \]

\[ B \cap C = \{e, f\} \]

\[ (A \cap B^c) \cup (B \cap C) = \{a, d, e, f\} \]
6. Let $U$ be the set of all TAMU students who are enrolled in Math 141 and let

\[ A = \{ x \in U | x \text{ is a female} \} \]
\[ B = \{ x \in U | x \text{ is an ECON major} \} \]
\[ C = \{ x \in U | x \text{ is a first-year student at TAMU} \} \]

(a) Describe in words the set $A^c \cap C$

The set of Math 141 students who are male and in their first year at TAMU.

(b) Use set notation to represent the set of Math 141 students who are female ECON majors but are not in their first year.

\[ A \cap B \cap C^c \]

7. Given $n(U) = 58$, $n(A) = 37$, $n(B) = 31$, and $n(A \cup B) = 49$, find $n(A \cap B^c)$.

\[
\begin{align*}
\text{n}(A \cup B) &= \text{n}(A) + \text{n}(B) - \text{n}(A \cap B) \\
49 &= 37 + 31 - \text{n}(A \cap B) \\
49 &= 68 - \text{n}(A \cap B) \\
-19 &= - \text{n}(A \cap B) \\
\text{n}(A \cap B) &= 19
\end{align*}
\]
8. A survey asked 100 people whether they like Whataburger, Chick-fil-A, and Jack in the Box.

- 57 people like Chick-fil-A.
- 49 people like Jack in the Box.
- 52 people like at least two of the restaurants.
- 18 people don’t like any of the restaurants.
- 42 people like both Whataburger and Chick-fil-A.
- 37 people like Chick-fil-A and Jack in the Box.
- 6 people like Chick-fil-A and Jack in the Box but don’t like Whataburger.

\[ a + b + c + d + e + f + g + h = 100 \]

a) How many people like Whataburger and Jack in the Box?
\[ d + e = 4 + 31 = 35 \]

b) How many people like Chick-fil-A but no other restaurants?
\[ c = 9 \]

c) How many people like Whataburger and Chick-fil-A but not Jack in the Box?
\[ b = 11 \]
9. Starbucks has 8 different lattes that each come in 3 different sizes. How many different orders are possible? 

\[ \mathbf{8 \cdot 3 = 24} \]

10. A computer password must consist of 6 characters. Each character can be either a lowercase letter or a digit (0-9).

(a) How many possible passwords are there?

\[ \mathbf{36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 = 36^6 = 2,176,782,336} \]

(b) How many passwords are possible if the password must start with a letter and no character can be repeated?

\[ \mathbf{26 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 = 1,012,851,840} \]

(c) What if, in addition, the password must alternate between letters and numbers?

\[ \mathbf{\frac{26 \cdot 10}{\text{let}} \cdot \frac{25 \cdot 9}{\text{let}} \cdot \frac{24 \cdot 8}{\text{let}} = 11,232,000} \]

11. Seven friends go to the movies. There are four females and three males.

(a) In how many ways can they arrange themselves in a single row?

\[ \mathbf{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5,040} \]

(b) In how many ways can they arrange themselves in a single row if a male must be sitting on each end?

\[ \mathbf{\frac{3 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2}{M} = 720} \]

(c) In how many ways can they arrange themselves in a single row if two of the females, Carol and Maggie, must sit next to each other?

1. Choose two seats for C and M  
2. Seat Carol and Maggie  
3. Seat the other 5  

\[ \mathbf{6 \cdot 2 \cdot 120 = 1,440} \]

12. An exam has 8 true/false questions and 14 multiple choice questions, each with 5 choices. How many ways can a student answer the exam if they answer all of the questions?

\[ \mathbf{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 2^8 \cdot 5^{14} = 1,562,500,000,000} \]

\[ \mathbf{1.5625 \times 10^{12}} \]