Math 141 Week-in-Review 6
Sections 6.4 and 7.1 and Review for Exam 2

Key Terms:

• **factorial**: For any natural number \( n \), \( n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 \)

• **permutation**: a permutation of a set of elements is an *ordered* arrangement of all the elements.

• **experiment**: an activity with observable results.

• **outcome**: the result of an experiment.

• **sample space**: the set of all outcomes of an experiment.

• **event**: a subset of the sample space. An event \( E \) is said to occur whenever \( E \) contains the observed outcome.

• **mutually exclusive**: Two events \( E \) and \( F \) are said to be mutually exclusive if \( E \cap F = \emptyset \).

Relevant Equations:

• The **number of permutations** of \( n \) distinct objects is given by \( P(n, n) = n! \)

• The **number of permutations of \( r \) distinct objects taken from a set of size \( n \)** is given by \( P(n, r) = \frac{n!}{(n-r)!} \)

• **Distinguishable Permutations**: Given a set of \( n \) objects in which \( n_1 \) objects are alike and of one kind, \( n_2 \) objects are alike and of another kind, ..., and \( n_m \) objects are alike and of yet another kind, so that \( n_1 + n_2 + \ldots + n_m = n \) then the number of distinguishable permutations of these \( n \) objects taken \( n \) at a time is given by \( \frac{n!}{n_1!n_2! \cdots n_m!} \).

• The **number of combinations of \( r \) distinct objects taken from a set of size \( n \)**, denoted by \( C(n, r) \), is given by \( C(n, r) = \frac{n!}{(n-r)!r!} \)

Connections between set theory and probability terminology:

• sample space \( \iff \) universal set

• event \( \iff \) set

• outcome \( \iff \) element

• impossible event \( \iff \) empty set

• certain event \( \iff \) universal set

• mutually exclusive \( \iff \) disjoint

**Note**: Union, intersection, and complement are all the same
1. A group of 12 people contains 7 males and 5 females. A photographer wants a picture of 7 of them: 4 men in the back row and 3 women in the front row. How many different arrangements can be made?

2. A convenience store has three different flavors of beef jerky. There are 8 bags of mild, 6 bags of hot, and 5 bags of teriyaki. If two bags of the same flavor can’t be distinguished, in how many distinguishable ways can the bags be arranged in a single row on the shelf?

3. A bag of M&M’s contains 6 blue M&M’s, 5 red M&M’s, and 3 yellow M&M’s.
   (a) In how many ways can a sample of 5 M&M’s be selected from the bag?
   (b) In how many ways can such a sample consist of exactly 2 red M&M’s?
   (c) In how many ways can the sample consist of at least 3 blue M&M’s?
   (d) In how many ways can the sample consist of exactly 2 red or exactly 3 blue M&M’s?
4. Adam has a collection of 10 DVDs. 5 of them are action movies, 3 are comedy, and 2 are drama. In how many ways can Adam arrange the 10 DVDs if each genre must stay together?

5. A congressional committee needs to be formed that includes 6 members. One of the members will be designated the chair and one the vice-chair. If this committee must be formed from a pool of 20 representatives, how many different committees can be formed?

6. An experiment consists of tossing a coin, rolling a six-sided die, and observing the outcomes.
   
   (a) Determine the sample space for this experiment.

   (b) Determine the event $E$ that a head is tossed and an odd number is rolled.

   (c) Determine the event $F$ that an even number is rolled.

   (d) Find $E \cup F$.

   (e) Are these two events mutually exclusive?

   (f) How many events does this experiment have?
Section 3.1: Graphing systems of inequalities

7. Determine graphically the solution set for the following system of inequalities. Indicate whether it is bounded or unbounded.

\[ 3x - 2y > 6 \]
\[ x + 2y \geq 14 \]
\[ x \leq 8 \]

Section 3.2: Setting up linear programming problems

8. A financier has earmarked at most $100,000 to invest in two projects. She estimates that project A will yield a return of 10% and project B will yield a return of 8%. Because project A has more risk than project B, she has decided to invest at least twice as much money in project B as she does in project A. How much should she invest in each project to maximize the total return on her investments? Set up the problem but do not solve.
9. Solve the following linear programming problem.

Maximize \( P = 5x + 3y \)
Subject to \( x - 5y \leq -5 \)
\( x + 3y \leq 15 \)
\( x + y \leq 7 \)
\( x \geq 0, y \geq 0 \)
Section 6.1: Sets and set operations

10. Consider the universal set $U = \{1, 2, 3, 4, a, b, c, d, e, f\}$ and the subsets $A = \{1, 2, 3, 4\}$, $B = \{x \mid x$ is a number that is even or a letter that is a vowel\}$, and $C = \{a, b, d, e\}$.

(a) TRUE   FALSE   \(2 \in A\)

(b) TRUE   FALSE   \(\{a, b\} \in C\)

(c) TRUE   FALSE   \(\{1, 4\} \subset A\)

(d) Write the set $B$ in roster notation.

(e) Find $A \cap B$.

(f) Find $(A \cup B) \cap C^c$.

(g) Are $A$ and $C$ disjoint?

11. Shade the appropriate regions in the following Venn Diagrams.

(a) $A \cup B^c \cup C^c$
(b) \((A \cup C^C) \cap B\)

Section 6.2: Number of elements in a set

12. A survey asked 130 people what flavors of Blue Bell ice cream they like. Let \(V\) be the set of people who like Homemade Vanilla. Let \(C\) be the set of people who like Cookies and Cream. Let \(M\) be the set of people who like Mint Chocolate Chip.

Use the survey results below to fill in the following Venn Diagram.

- 75 people like Homemade Vanilla.
- 89 people like Cookies and Cream.
- 93 people like at least two of the three flavors.
- 49 people like Homemade Vanilla and Mint Chocolate Chip.
- 56 people like Mint Chocolate Chip and Cookies and Cream.
- 11 people only like Mint Chocolate Chip.
- 31 people like all three flavors.
(a) How many people don’t like any of the flavors?

(b) How many people like exactly one flavor?

Section 6.3: Multiplication principle

13. A license plate consists of three letters followed by three numbers (0-9). How many different license plates can be formed that start with an R and do not repeat letters or numbers?

14. A group of five males and four females go to the movies. In how many ways can they sit in a single row of nine seats if they must alternate gender?