Math 141 Week-in-Review 7
Sections 7.2-7.4

Key Terms:

- **relative frequency**: Suppose we repeat an experiment \( n \) times and an event \( E \) occurs \( m \) of those times. Then \( \frac{m}{n} \) is called the relative frequency of the event \( E \).

- **empirical probability**: If the relative frequency approaches a certain value as \( n \) becomes larger and larger, we call this value the empirical probability.

- **probability**: The probability of an event is a number that lies between 0 and 1, inclusive, that represents the likelihood of the event occurring. The larger the probability, the more likely the event is to occur.

- **simple (or elementary) event**: an event that consists of exactly one outcome

- **probability distribution**: a table that lists the probability of each simple event of an experiment

- **probability function**: the function which assigns a probability to each simple event of an experiment

- **uniform sample space**: a sample space in which all of the outcomes are equally likely

Relevant Equations:

- For a uniform sample space \( S = \{s_1, s_2, ..., s_n\} \), \( P(s_1) = P(s_2) = \cdots = P(s_n) = \frac{1}{n} \).

- Let \( S \) be a uniform sample space and let \( E \) be any event. Then,
  \[
P(E) = \frac{\text{Number of outcomes in } E}{\text{Number of outcomes in } S} = \frac{n(E)}{n(S)}
\]

Properties of Probability Functions:

1. \( 0 \leq P(s_i) \leq 1 \)
2. \( P(s_1) + P(s_2) + \cdots + P(s_n) = 1 \)
3. \( P(\{s_i\} \cup \{s_j\}) = P(s_i) + P(s_j) \) for \( i \neq j \)

Finding the Probability of an Event \( E \):

1. Determine a sample space \( S \) associated with the experiment.
2. Assign probabilities to the simple events of \( S \).
3. If \( E = \{s_1, ..., s_m\} \) where \( \{s_1\}, ..., \{s_m\} \) are simple events then
  \[
P(E) = P(s_1) + P(s_2) + \cdots + P(s_m)
\]
4. If \( E \) is the empty set, then \( P(E) = 0 \).

Rules of Probability:

1. \( P(E) \geq 0 \) for any event \( E \).
2. \( P(S) = 1 \).
3. If \( E \) and \( F \) are mutually exclusive (that is, \( E \cap F = \emptyset \)), then \( P(E \cup F) = P(E) + P(F) \).
4. For any events \( E \) and \( F \), \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \).
5. \( P(E^c) = 1 - P(E) \).
1. Consider the experiment of flipping a coin and noting whether it lands heads or tails then rolling a six-sided die and noting the uppermost facing number.

(a) List the simple events associated with this experiment.
(b) What is the probability that a heads is flipped?
(c) What is the probability that a tails is flipped and an even number is rolled?

2. (Tan 7.2 #5) In a survey conducted to determine whether movie attendance is increasing (i), decreasing (d), or holding steady (s) among various sectors of the population, participants are classified as follows:
   Group 1: Those aged 10-19
   Group 2: Those aged 20-29
   Group 3: Those aged 30-39
   Group 4: Those aged 40-49
   Group 5: Those aged 50 and older

The response and age group of each participant are recorded. List the simple events associated with this experiment.

3. A six-sided die is weighted so that the probability of rolling each number is given by the table below.

<table>
<thead>
<tr>
<th>Roll</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$\frac{1}{10}$</td>
<td>$\frac{1}{10}$</td>
<td>$\frac{2}{10}$</td>
<td>$\frac{3}{10}$</td>
<td>$\frac{2}{10}$</td>
<td>$\frac{1}{10}$</td>
</tr>
</tbody>
</table>

(a) What is the probability of rolling a 2 or a 5?
(b) What is the probability of rolling an odd number?

4. (Tan 7.2 #16) The accompanying data were obtained from a survey of 1500 Americans who were asked: How safe are American-made consumer products?

<table>
<thead>
<tr>
<th>Rating</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Respondents</td>
<td>285</td>
<td>915</td>
<td>225</td>
<td>30</td>
<td>45</td>
</tr>
</tbody>
</table>

A: Very safe
B: Somewhat safe
C: Not too safe
D: Not safe at all
E: Don’t know

(a) Determine the empirical probability distribution associated with these data.
(b) If a person who participated in the survey is selected at random, what is the probability that they responded with an A or a B?
5. Determine whether the sample spaces associated with the given experiments are uniform.

(a) A card is drawn at random from a standard 52-card deck and the suit is observed.
(b) A ball is selected at random from an urn containing 5 red balls and 7 green balls, and the color is observed.
(c) Two fair six-sided dice are rolled and the number appearing uppermost on each die is observed.
(d) Two fair six-sided dice are rolled and the sum of the numbers appearing uppermost is observed.

6. Let $E$ and $F$ be two events of an experiment with sample space $S$. Suppose $P(E) = 0.4$, $P(F) = 0.7$ and $P(E \cup F) = 0.8$. Compute:

a) $P(E \cap F)$

b) $P(E^C)$

c) $P(E^C \cap F)$

7. Let $E$ and $F$ be mutually exclusive events and suppose $P(E) = 0.3$ and $P(F) = 0.25$. Compute:

a) $P(E \cup F)$

b) $P(F^C)$

c) $P(E^C \cap F^C)$

8. A card is drawn at random from a standard 52-card deck. Find the probability of the following events:

a) $E$ is the event that a jack or a heart is drawn.

b) $F$ is the event that a queen or a king is drawn.

c) $G$ is the event that a 2 or 3 is not drawn.

9. A pair of fair six-sided dice are rolled and the numbers facing uppermost on each die are observed. Find the probability of the following events:

a) $E$ is the event that the sum of the dice is not 5.

b) $F$ is the event that at least one of the dice is a 4 or at least one of the dice is a five.

10. A test has 8 true/false questions. If a student randomly guesses on every question, what is the probability that they will get at least 4 problems correct?

11. A test has 8 multiple choice questions with 4 choices each. If a student randomly guesses on every question, what is the probability that they will get exactly 4 problems correct?
12. A test has 4 true/false questions and 4 multiple choice questions with 4 choices each. If a student randomly guesses on every question, what is the probability that they will get exactly 2 true/false problems correct and exactly 2 multiple choice questions correct?

13. A five card hand is dealt from a standard 52-card deck.
   a) What is the probability that the hand contains a three-of-a-kind (three cards of the same rank)?
   b) What is the probability that the hand is a full house (three cards of one rank and two of another)?
   c) What is the probability that exactly three of the cards are red?

14. A bag of 23 Skittles contains 5 red Skittles, 4 yellow Skittles, 3 green Skittles, 5 orange Skittles, and 6 purple Skittles. A sample of 7 Skittles is chosen from the bag.
   a) What is the probability that the sample contains exactly 2 purple Skittles?
   b) What is the probability that the sample contains at least 1 purple Skittle?
   c) What is the probability that the sample contains exactly 2 red Skittles or exactly 2 yellow Skittles?