Math 141 Week-in-Review 8

Sections 7.5-7.6

Key Terms and Relevant Equations:

- The **conditional probability** that the event \( F \) will occur given that the event \( E \) occurs is
  \[
P(F|E) = \frac{P(E \cap F)}{P(E)}.
  \]

- **Product Rule:** \( P(E \cap F) = P(E)P(F|E) \).

- **Independent:** Two events \( E \) and \( F \) are said to be independent if \( P(E|F) = P(E) \) and \( P(F|E) = P(F) \). In other words, two events are independent if the outcome of one does not affect the outcome of the other.

- Two events \( E \) and \( F \) are independent if \( P(E \cap F) = P(E)P(F) \)

- \( n \) events \( E_1, E_2, ..., E_n \) are independent if \( P(E \cap E_2 \cap \cdots \cap E_n) = P(E_1)P(E_2)\cdots P(E_n) \)

- **Bayes’ Theorem:**
  \[
P(E|F) = \frac{P(E)P(F|E)}{P(E)P(F|E) + P(E^c)P(F|E^c)}
  \]

1. A survey of 240 people asked people from different age groups how many movies they had seen in theaters in the last month. The results are in the table below.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>0 movies</th>
<th>1 movie</th>
<th>2 movies</th>
<th>3 or more</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
<td>23</td>
<td>18</td>
<td>21</td>
<td>9</td>
<td>71</td>
</tr>
<tr>
<td>18-35</td>
<td>24</td>
<td>39</td>
<td>14</td>
<td>2</td>
<td>94</td>
</tr>
<tr>
<td>Over 35</td>
<td>44</td>
<td>24</td>
<td>5</td>
<td>2</td>
<td>75</td>
</tr>
<tr>
<td>Total</td>
<td>91</td>
<td>81</td>
<td>40</td>
<td>28</td>
<td>240</td>
</tr>
</tbody>
</table>

a) If one of the surveyed people is chosen at random, what is the probability that they saw 1 movie in the last month?

\[
\frac{81}{240} = 0.3375
\]

b) If you know that the person is 18-35, what is the probability that they saw 1 movie in the last month?

\[
P(1 \text{ movie} | 18-35) = \frac{P(1 \text{ movie} \cap 18-35)}{P(18-35)} = \frac{39}{94} = \frac{39}{94}
\]

c) What is the probability that a person is over 35 given that they saw 2 or more movies?

\[
\frac{7}{68} = \frac{over \ 35 \ \text{and saw 2 or more}}{saw \ 2 \ or \ more}
\]
2. A pair of fair six-sided dice are rolled.

(a) What is the probability that the sum of the numbers falling uppermost is odd if it is known that one of the numbers is a 2?

\[
\begin{array}{c}
1 & 2 & 3 & 4 & 5 & 6 \\
1 & \times & \times & \times & \times & \times \\
2 & \times & \times & \times & \times & \times \\
3 & \times & \times & \times & \times & \times \\
4 & \times & \times & \times & \times & \times \\
5 & \times & \times & \times & \times & \times \\
6 & \times & \times & \times & \times & \times \\
\end{array}
\]

we roll a 2 and sum is odd

\[
\begin{array}{c}
(1,1) & (2,2) & (3,3) & (4,4) & (5,5) & (6,6) \\
1 & 1 & 1 & 1 & 1 & 1 \\
\end{array}
\]

we roll a 2.

\[
\frac{4}{10} = \frac{2}{5}
\]

(b) What is the probability that the sum of the numbers falling uppermost is at least 6 if it is known that a double (two of the same number) is rolled?

\[
\begin{aligned}
\{ & (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \\
\end{aligned}
\]

3. A certain math class has 40% male students and 60% female students. 30% of the females got an A on the first exam and 25% of the males got an A on the first exam.

(a) What is the probability that a student chosen at random is a male who got an A?

Product Rule:
\[
P(E \cap F) = P(E) \cdot P(F|E)
\]

\[
\begin{aligned}
& P(M) \cdot P(A|M) = .4 \cdot .25 = .1 \\
& P(F) \cdot P(A|F) = .6 \cdot .3 = .18
\end{aligned}
\]

(b) What is the probability that a student chosen at random is a female who did not get an A?

\[
.6 \cdot .7 = .42
\]

(c) What is the probability that a student chosen at random got an A?

\[
.4 \cdot .25 + .6 \cdot .3 = .1 + .18 = .28
\]
4. Three cards are drawn without replacement from a standard 52-card deck.

(a) What is the probability that all three cards are spades? (Can you figure this out two ways?)

\[ P(3 \text{ spades}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{11}{850} \]

Total # of 3 card hands: \(C(52, 3)\)

# of 3 spade hands: \(C(13, 3)\)

\[ \frac{C(13, 3)}{C(52, 3)} = \frac{286}{22100} = \frac{11}{850} \]

(b) What is the probability that the first two cards are spades but the third one is not?

\[ \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{39}{50} = \frac{39}{850} \]

(c) What is the probability that the last card is a spade?

\[ \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} + \frac{13}{52} \cdot \frac{39}{51} \cdot \frac{12}{50} + \frac{39}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} + \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{13}{50} = \frac{1}{4} \]

5. A box contains 8 batteries of which two are dead. The batteries are selected one at a time and without replacement until a battery is found which is not dead. What is the probability that the number of batteries tested is (a) One? (b) Two? (c) Three?

(a) \( P(1) = \frac{6}{8} = \frac{3}{4} \)

(b) \( P(2) = \frac{2}{8} \cdot \frac{6}{7} = \frac{12}{56} = \frac{3}{14} \)

(c) \( P(3) = \frac{2}{8} \cdot \frac{1}{7} \cdot 1 = \frac{2}{56} = \frac{1}{28} \)
6. A card is drawn from a standard 52-card deck. Let $E$ be the event that a red card is drawn. Let $F$ be the event that an ace is drawn.

a) Are $E$ and $F$ independent?

$$P(E) = \frac{26}{52} = \frac{1}{2} \quad P(F) = \frac{4}{52} = \frac{1}{13} \quad P(E \cap F) = \frac{2}{52} = \frac{1}{26}$$

$$P(E) \cdot P(F) = \frac{1}{2} \cdot \frac{1}{13} = \frac{1}{26} \quad \text{Yes}$$

b) Are $E$ and $F$ mutually exclusive?

No because $E \cap F \neq \emptyset$.

7. Let $E$ and $F$ be independent events such that $P(E) = .4$ and $P(F) = .5$. Find:

(a) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$$= .4 + .5 - .4 \cdot .5$$

$$= .7$$

(b) $P(E^C \cap F^C) = P((E \cup F)^C)$

$$= 1 - P(E \cup F)$$

$$= 1 - .7 = .3$$

8. Let $E$ and $F$ be mutually exclusive events such that $P(E) = .4$ and $P(F) = .5$. Find:

(a) $P(E \cup F) = .4 + .5 = .9$

(b) $P(E^C \cap F^C) = .1$

9. There is a 40% chance that it will rain tomorrow (Monday), a 40% chance that it will rain on Tuesday, and a 75% chance that it will rain on Wednesday.

(a) Assuming that the likelihood that it will rain each day is independent, what is the probability that it rains all three days?

$$P(M \cap T \cap W) = .4 \cdot .4 \cdot .75 = .12$$

$$P(M) \cdot P(T) \cdot P(W)$$

(b) What is the probability that it doesn’t rain any of the days?

$$P(M^C \cap T^C \cap W^C) = .6 \cdot .6 \cdot .25 = .09$$
10. A certain math class has 40% male students and 60% female students. 30% of the females got an A on the first exam and 25% of the males got an A on the first exam. What is the probability that a student is a female if we know that they got an A?

\[
P(F | A) = \frac{P(F \cap A)}{P(A)}
\]

\[
= \frac{0.6 \cdot 0.3}{0.4 \cdot 0.25 + 0.6 \cdot 0.3} = \frac{9}{14}
\]

11. At a certain Aggie football game, there are 100,000 people in attendance. Of those people, 70,000 are students and 30,000 are not. During the game, 20% of the students buy concessions and 60% of the non-students buy concessions. A person at the game is selected at random.

a) What is the probability that the person is a student who bought concessions?

\[0.7 \cdot 0.2 = 0.14\]

b) What is the probability that the person did not buy concessions?

\[0.7 \cdot 0.8 + 0.3 \cdot 0.4 = 0.56 + 0.12 = 0.68\]

c) What is the probability that the person is a student given that they did not buy concessions?

\[P(S | C^c) = \frac{P(S \cap C^c)}{P(C^c)} = \frac{0.7 \cdot 0.8}{0.68} = \frac{0.56}{0.68} = \frac{14}{17}\]
12. There is a bag of M&M's that contains 6 red M&M's, 4 green M&M's, and 5 blue M&M's. Two M&M's are chosen from the bag at random, without replacement.

a) Given that the first M&M is green, what is the probability that the second M&M is red?

\[
\frac{6}{14} = \frac{3}{7}
\]

b) What is the probability that both M&M's chosen are green?

\[
\frac{4}{15} \cdot \frac{3}{14} = \frac{12}{210} = \frac{2}{35}
\]

c) If the second M&M chosen was red, what is the probability that the first one was blue?

\[
P(B_1 \mid R_2) = \frac{P(B_1 \cap R_2)}{P(R_2)} = \frac{\frac{5}{15} \cdot \frac{6}{14}}{\frac{6}{15} \cdot \frac{5}{14} + \frac{4}{15} \cdot \frac{6}{14} + \frac{5}{15} \cdot \frac{6}{14}}
\]

\[
= \frac{5}{14}
\]
13. Use the following information to complete the tree diagram and answer the questions that follow. \( P(A) = 0.4, P(B) = 0.2, P(E|A) = .2, P(D|B) = .6, \) and \( P(E|C) = .6. \)

a) Find \( P(E). \)

\[
0.4 \cdot 0.2 + 0.2 \cdot 0.4 + 0.4 \cdot 0.6 = 0.4
\]

b) Find \( P(C \cup E). \)

\[
P(C \cup E) = P(C) + P(E) - P(C \cap E)
= 0.4 + 0.4 - 0.4 \cdot 0.6 = 0.56
\]

c) Find \( P(B|D). \)

\[
P(B|D) = \frac{P(B \cap D)}{P(D)} = \frac{0.2 \cdot 0.6}{1 - P(E)} = \frac{0.12}{1 - 0.4} = 0.2
\]

d) Are \( E \) and \( B \) independent events?

\[
P(E) = 0.4 \quad P(B) = 0.2
\]

\[
P(E \cap B) = 0.2 \cdot 0.4
\]

Yes