Teaching Statement for Christopher J. Hillar

I began teaching mathematics in 1997 at the Southwest Texas State University summer honors math program for gifted high school students. At the time, I was a freshman at Yale. The experience taught me some valuable lessons. For one, I learned that there is no uniform methodology that applies to all students of mathematics, even when taking ability into account. For instance, while some of the brightest in my group required only a little intuition to get started on a problem or new concept, other students needed to experiment with the objects first in order to develop that intuition. At first this made my job challenging, but soon enough I began to deliver my material in as many ways as I felt there were ways of understanding it. Of course, this is made even more difficult when it all must happen in one lecture. Another important discovery I made here was that not everyone knows how engaging and important mathematics can be. The fundamental problems and theorems in math are almost always natural and arise from questions of basic understanding. By explaining some of this history and sense of discovery–and that these are human endeavors–it is easier to motivate students to ask natural questions themselves. These simple lessons form a fundamental component of how I teach mathematics.

Aside from being a teaching assistant for a linear algebra course at the University of California, Berkeley, I have taught Calculus II at Texas A&M for both science majors (Math 172) and engineering majors (Math 152). My teaching approach in both classes was the same, although the level of the presentation was necessarily higher in the first one (Math 172). Although much has changed since the days of my mentoring in 1997 and 1998, the discoveries outlined above have not. The basic structure of each of my lectures was as follows. First, I motivate the topic by asking a basic question that arises naturally. This is followed by a discussion of how one might use intuition to see what should be a definition or concept. This leads me to a general theorem, which I then check by verifying if it implies what it should for the original motivating problem. For instance, consider my beginning lecture on area and integration. It is natural to want to define the concept of the "area" of a shape. A square with side length 1, for instance, should have area 1. In fact, why not simply assert this to be the case? Next, we would want areas to add when shapes are placed next to one another; moreover, area scaling should, well, scale area. This forces the areas of all rectangles to be determined. How about a right triangle? Well, that is half of a rectangle, so that is easy. However, what are we to do with a circle? That is where the natural idea of exhaustion comes in. Can we approximate this circle with rectangles? If so, does the limit of these approximations make sense? If so, can we actually compute its value? After developing the general theory, I then come back to the motivating problems and make sure that the answers I get make sense. As another more metaphysical example, I present the story of Zeno's paradox to explain how the concept of "infinite sum" could occur naturally, even to a philosopher (or a turtle). If one wanted to make sense of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$, how else would one do it other than compute the sequence of partial sums and see if it has a limit? In this way, I try and convey the message that one can be lead to definitions and concepts by pure thought and not just serendipity.

Of course, I have no illusions that this will work with everyone. Therefore, I always make sure that I present several examples, methodically illustrating the steps in the algorithms that will solve Calculus II problems. So far I have been very encouraged by how involved my students are in my classes, and I look forward to a future of teaching mathematics.