The parabolic Harnack inequality for integro-differential operators

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We generalize the elliptic Harnack inequality proved in [1]. Additionally we relax the assumptions of [1]: the integral term measure in it is absolutely continuous with respect to the Lebesgue measure. The proof exploits the basic construction of Krylov-Safonov for elliptic operators and its modernization for the purpose of adding integral terms by R. Bass.

Let $B(x,r)$ denote $\{y \in \mathbb{R}^d : |y-x| < r\}$, Q be the cylinder $\{t \in [0,2),\ |x| \in B(0,1)\}$, and W be the space of Borel measurable bounded non-negative functions on $\mathbb{R}^{d+1}$ with restriction on Q belonging to the Sobolev space $W^{1,2}(Q)$. And let $\mathcal{L}$ be an operator with Borel measurable coefficients, acting on $u \in W$ according to the formula

$$\begin{aligned}\mathcal{L}u(t,x) &= \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(t,x)\frac{\partial^2 u(t,x)}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(t,x)\frac{\partial u(t,x)}{\partial x_i} \\ &+ \int_{\mathbb{R}^d\setminus\{0\}} [u(t,x+h) - u(t,x) - 1_{(|h|\le 1)} h \cdot \nabla u(t,x)]\mu(t,x;dh).\end{aligned}$$

ASSUMPTIONS There exist positive constants λ, K, k and β such that for all t, x:

1) $\lambda |y|^2 \le y^T a(t,x) y,\ y \in \mathbb{R}^d$;

2) $\|a(t,x)\| + |b(t,x)| + \int_{\mathbb{R}^d} (|h|^2 \wedge 1)\mu(t,x;dh) \le K$;

3) for any $r \in (0,1]$, $y_1,\ y_2 \in B(x,r/2) \cap B(0,1)$ and borel A with $\mathrm{dist}(x,A) \ge r$ holds $\mu(t,y_1;\{h : y_1 + h \in A\}) \le k r^{-\beta} \mu(t,y_2;\{h : y_2 + h \in A\})$.

Theorem 1 *There exists a positive constant $C(d,\ \lambda,\ K,\ k,\ \beta)$ s.t. for every $u \in W$ satisfying*

$$\frac{\partial}{\partial t} + \mathcal{L}u = 0 \ \textit{a.e. in}\ (t,x)$$

holds: for all $|x| \le \frac{1}{2}$

$$u_{0,x} \ \ge\ C u_{1,0}.$$

The crucial point of the proof is the extension of Prop. 3.9 [1] to the cylinder.

References

[1] Foondun, M. *Harmonic functions for a class of integro-differential operators.* Potential Anal., 31, 2009, 21-44.

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