Relativistic point dynamics and Einstein's formula as a property of localized solutions of a nonlinear Klein-Gordon equation

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Relativistic mechanics includes the relativistic dynamics of a mass point and the relativistic field theory. In a relativistic field theory the relativistic field dynamics is derived from a relativistic covariant Lagrangian, such a theory allows to define the total energy and momentum, forces and their densities but does not provide a canonical way to define the mass, position or velocity for the system. For a closed system *without external forces* the total momentum has a simple form $\mathbf{P} = M\mathbf{v}$ where $\mathbf{v}$ is a *constant velocity*, allowing to define naturally the total mass M and to derive from the Lorentz invariance Einstein's energy-mass relation $E = Mc^2$ with $M = m_0\gamma$, with γ the Lorentz factor and m_0 the rest mass; according to Einstein's formula the rest mass is determined by the internal energy of the system.

The relativistic dynamics of a mass point is described by a relativistic version of Newton's equation where the rest mass m_0 of a point is prescribed; in Newtonian mechanics the mass M reveals itself in *accelerated motion* as a measure of inertia which relates the point acceleration to the external force. The question which we address is the following: Is it possible to construct a mathematical model where the internal energy of a system affects its acceleration in an external force field as the inertial mass does in Newtonian mechanics?

We construct a model which allows to consider in the same framework the uniform motion in the absence of external forces (a closed system) and the accelerated motion caused by external fields; the internal energy is present both in uniform and accelerating regimes. The model is based on the nonlinear Klein-Gordon (KG) equation which is a part of our theory of distributed charges interacting with electromagnetic (EM) fields, [1]-[4]. We prove that if a sequence of solutions of a KG equation concentrates at a trajectory $\mathbf{r}(t)$ and their local energies converge to $E(t)$ then the trajectory satisfies the relativistic version of Newton's equation where the mass is determined in terms of the energy by Einstein's formula, and the EM forces are determined by the coefficients of the KG equation. We prove that the concentration assumptions hold for the case of a general rectilinear accelerated motion.

References

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