On the propagation of Monokinetic Measures with Rough Momentum Profile

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This is a report on a work in progress jointly with Francois Golse, Peter Markowich and Thierry Paul.

The analysis of the global flow defined by the Hamiltonian system

$$\begin{split} \dot{X}_t &= \nabla_{\xi} H(X_t, \Xi_t) \quad X_0(x, \Xi) = x \\ \dot{\Xi}_t &= \nabla_x H(X_t, \Xi_t) \quad \Xi_0(x, \xi) = \nabla_x U(x) \end{split}$$

is a standard tool in the WKB asymptotics.

In the present contribution it will be interpreted as the propagation of a monokinetic measure: The push forward by the Hamiltonian flow of a measure of the form:

$$\mu(x,\xi) = \rho^{\mathrm{in}}(x)\delta_{U(x)}(\xi)$$

evolving according to the Liouville equation:

$$\partial_t \mu + \{H, \mu\} = 0.$$

This approach leads us to an estimate of the number of folds of the Lagrangian Manifold even for rough initial data. We also provide informations on the structure of the push-forward $\rho(t, x)$ measure under the canonical projection of the space $\mathbb{R}_x \times \mathbb{R}_{\xi}$ on \mathbb{R}_x .