Inertial manifolds for strongly damped wave equations

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Consider the boundary value problem for a semilinear strongly damped wave equation in a bounded domain Ω:

$$u_{tt} - 2\gamma\Delta u_t = \Delta u + f(u), \quad u\big|_{\partial\Omega} = 0, \tag{1}$$

$$u\big|_{t=0} = u_0(x) \in H_0^1(\Omega), \quad u_t\big|_{t=0} = p_0(x) \in L_2(\Omega). \tag{2}$$

Here $\gamma > 0$ is a coefficient of strong dissipation, and the nonlinearity $f(u)$ satisfies the global Lipschitz condition:

$$|f(v_1) - f(v_2)| \leqslant L\,|v_1 - v_2| \qquad \forall v_1, v_2 \in \mathbb{R}.$$

The following theorem gives sufficient condition for the existence of an inertial manifold for equation (1).

Theorem 1 *Let λ_k, $0 < \lambda_1 < \lambda_2 \leqslant \dots \to +\infty$, be eigenvalues of the operator $-\Delta$ in the domain Ω under the Dirichlet boundary conditions. Suppose that there is an N such that the following inequalities hold*

$$\lambda_N < \lambda_{N+1} < 1/(2\gamma^2), \tag{3}$$

$$2L < \sup_{\gamma\lambda_N \leqslant \Phi < \gamma\lambda_{N+1}} \{(\gamma\lambda_{N+1} - \Phi)\min\{\varkappa_1(\Phi), \varkappa_N(\Phi), \varkappa_{N+1}(\gamma\lambda_{N+1})\}\}, \tag{4}$$

where we have used the notation

$$\varkappa_k(\Phi) = \Phi - \gamma\lambda_k + \sqrt{\Phi^2 - 2\gamma\lambda_k\Phi + \lambda_k}.$$

Then, in the phase space $H_0^1(\Omega) \times L_2(\Omega)$, there exists a $2N$-dimensional inertial manifold that exponentially attracts (as $t \to +\infty$) all the solutions of problem (1), (2).

The proof is based on the construction of a new inner product in the phase space in which gap property holds and thus an inertial manifold exists (the corresponding general theorem for an abstract differential equation in a Hilbert space one can find, e.g., in [1]).

Remark. If γ, λ_N and λ_{N+1} are fixed and they satisfy (3), then we state the existence of an inetial manifold for sufficiently small L.

References

[1] A. Yu. Goritskii and V. V. Chepyzhov, *The Dichotomy Property of Solutions of Quasilinear Equations in Problems on Inertial Manifolds*, Mat. Sb. **196** (2005), no. 4, 23–50.

[2] N. A. Chalkina, *Sufficient Condition for the Existence of an Inertial Manifold for a Hyperbolic Equation with Weak and Strong Dissipation*, Russ. J. Math. Phys. **19** (2012), 11–20.