Trajectory attractors for equations of mathematical physics

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We describe the method of trajectory dynamical systems and trajectory attractors and we apply this approach to the study of the limiting asymptotic behaviour of solutions of non-linear evolution equations. This method is especially useful in the study of dissipative equations of mathematical physics for which the corresponding Cauchy initial-value problem has a global (weak) solution with respect to the time but the uniqueness of this solution either has not been established or does not hold. An important example of such an equation is the 3D Navier–Stokes system in a bounded domain (see [1]). In such a situation one cannot use directly the classical scheme of construction of a dynamical system in the phase space of initial conditions of the Cauchy problem of a given equation and find a global attractor of this dynamical system. Nevertheless, for such equations it is possible to construct a trajectory dynamical system and investigate a trajectory attractor of the corresponding translation semigroup.

This universal method is applied for various types of equations arising in mathematical physics: for general dissipative reaction-diffusion systems, for the 3D Navier–Stokes system, for dissipative wave equations, for non-linear elliptic equations in cylindrical domains, and for other equations and systems. Special attention is given to using the method of trajectory attractors in approximation and perturbation problems arising in complicated models of mathematical physics.

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References

[1] V.V. Chepyzhov, M.I. Vishik, *Attractors for Equations of Mathematical Physics* Amer. Math. Soc. Colloq. Publ., **49**, Amer.Math. Soc., Providence, RI, 2002.