Relative version of the Titchmarsh convolution theorem

EVGENY GORIN

Moscow State Pedagogical University, Moscow, Russia evgeny.gorin@mtu-net.ru

> DMITRY TRESCHEV Steklov Mathematical Institute treschev@mi.ras.ru

We consider the algebra $C_u = C_u(\mathbb{R})$ of uniformly continuous bounded complex functions on the real line \mathbb{R} with pointwise operations and sup-norm. Let I be a closed ideal in C_u invariant with respect to translations, and let $\mathrm{ah}_I(f)$ denote the minimal real number (if it exists) satisfying the following condition. If $\lambda > \mathrm{ah}_I(f)$, then $(\hat{f} - \hat{g})|_V = 0$ for some g_I , where V is a neighborhood of the point λ . The classical Titchmarsh convolution theorem is equivalent to the equality $\mathrm{ah}_I(f_1 \cdot f_2) = \mathrm{ah}_I(f_1) + \mathrm{ah}_I(f_2)$, where $I = \{0\}$. We show that, for ideals I of general form, this equality does not generally hold, but $\mathrm{ah}_I(f^n) = n \cdot \mathrm{ah}_I(f)$ holds for any I. We present many nontrivial ideals for which the general form of the Titchmarsh theorem is true.