Incompressible limit of the linearized Navier–Stokes equations

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We consider initial–boundary value problem for linearized equations of viscous barotropic fluid motion in a bounded domain. We briefly discuss results on existence, uniqueness and estimates of weak solutions to this problem (see [1, 2, 3]). Then we focus on the asymptotic behaviour of the solutions as the compressibility tends to zero, i.e. on the passage to so-called *incompressible limit* (see [4, 5]). Briefly, we show that

- in general case the velocity field converges *weakly* in $L^2(0,T;H^1_0)$;
- if the initial condition for the velocity is divergence-free then the velocity converges *strongly* and the pressure converges $*$-*weakly* in $L^\infty(0,T;L^2)$;
- if, in addition, the *initial condition* for the pressure is compatible with the *initial value* of the pressure in the incompressible problem then the convergence of the pressure is *strong*. (A similar compatibility condition was obtained in [6] as a *necessary condition* of strong convergence of the solutions.)

We also demonstrate the necessity of these sufficient conditions using explicit solutions which are available for simplified data.

References

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