On a compactness problem

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Let V, H be two real Hilbert spaces, $V \subset H$ with compact and dense imbedding and let $A : V \to V'$ be bounded, self ajoint and coercive. Under reasonable assumptions on g , a compactness property of the range is established for energy-bounded solutions of an abstract equation

$$u'' + Au + g(u') = h(t), \quad t \geq 0$$

when h is S^1-uniformly continuous with values in H. This property allows to deduce :

1) asymptotic almost-periodicity of all solutions of wave or plate equations in presence of an almost-periodic source term when the damping term is strong enough.

2) convergence to equilibrium of all solutions of some equations of the form

$$u'' + Au + f(u) + g(u') = h(t), \quad t \geq 0$$

when f is the gradient of a potential satisfying the Łojasiewicz gradient inequality, g is sufficiently coercive globally and $h(t) \to 0$ sufficiently fast for t tending to infinity.

These results generalize, mainly to the case of non-local damping terms, some previous works by the author and his colleagues and rely on methods developped during more than 30 years, cf. e.g. [1, 2, 3] for applications of type 1) and [4, 5] for applications of type 2). There are still challenging open problems in this direction which will be mentionned during the lecture.

References

[1] L. Amerio & G. Prouse, *Uniqueness and almost-periodicity theorems for a non linear wave equation*, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. **8**, 46 (1969) 1–8.

[2] M. Biroli, & A. Haraux, *Asymptotic behavior for an almost periodic, strongly dissipative wave equation.* J. Differential Equations **38** (1980), no. 3, 422–440.

[3] A. Haraux, *Damping out of transient states for some semi-linear, quasi-autonomous systems of hyperbolic type*, Rc. Accad. Naz. Sci. dei 40 (Memorie di Matematica) **101** 7, fasc.7 (1983), 89–136.

[4] A. Haraux & M.A. Jendoubi, *Convergence of bounded weak solutions of the wave equation with dissipation and analytic nonlinearity.* Calc. Var. Partial Differential Equations **9** (1999), no. 2, 95–124.

[5] I. Ben Hassen & L. Chergui, *Convergence of global and bounded solutions of some nonautonomous second order evolution equations with nonlinear dissipation*, J. Dynam. Differential Equations **23**(2011), no. 2, 315–332.