Bony and thick attractors

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Understanding of the structure of attractors of generic dynamical systems is one of the major goals of the theory. A vast general program suggested by Palis presents numerous conjectures about this structure. Various particular cases of these conjectures are proved in numerous papers that we do not quote here. Main part of these investigations is related to diffeomorphisms of closed manifolds. Our investigation is parallel to this direction of research. In the first part of the talk, attractors of *manifolds with boundary onto themselves* are studied. At present, locally generic properties of attractors of such maps are established, that are not yet observed (and plausibly do not hold) for the case of closed manifolds. For instance, an open set of diffeomorphisms of manifolds with boundary onto themselves may have attractors with intermingled basins. The strongest result of this kind is in [3].

Another property of this kind is *having thick attractors*. It is a general belief that attractors of typical smooth dynamical systems (diffeomorphisms and flows) on closed manifolds either coincide with the whole phase space or have Lebesgue measure zero. In this talk we show that this is not the fact for diffeomorphisms of manifolds with boundary onto themselves. Namely, in the space of diffeomorphisms of $T^2 \times [0,1]$, there exists an open set such that any map from a complement of this set to a countable number of hypersurfaces, has a thick attractor: a transitive attractor that has positive Lebesgue measure together with its complement [1]. The problem whether thick attractors exist for locally generic diffeomorphisms of a closed manifold remains widely open.

In the second part we study so called bony attractors. These are attractors of skew products over a Bernoulli shift with the following unexpected property: the map has an invariant manifold, and the intersection of the attractor with that manifold, called *a bone*, is much larger than the attractor of the restriction of the map to the invariant manifold. Bony attractors with one-dimensional bones were discovered in [4]. We construct bones of arbitrary dimension [2]. It is expected that bony attractors are in a sense locally generic.

References

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[4] Yu. G Kudryashov, *Bony attractors*, Func. Anal. Appl. **44** (2010), no. 3, 73–76.