On the uniform attractors of finite-difference schemes

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The theory of global uniform attractors of non-autonomous differential systems has been constructed in [1]. It is important in applications how close the attractors of discrete approximations to mathematical models are to their true attractors. For autonomous equations, this problem was studied in [2], where a theorem on the semicontinuous dependence of attractors of a family of semidynamical systems on the parameter was proved. A similar result was obtained in [3] for uniform attractors of families of semiprocesses corresponding to non-autonomous evolution equations. It was assumed in [2, 3] that the considered families have a common time semigroup; therefore, when studying finite-difference, the grid increment was represented in the form $\tau = \tau_n = T_0/n$ where T_0 is some positive number and $n \in \mathbb{N}$. In this paper we prove a theorem on the upper semi-continuous dependence on the parameter of the uniform attractors of families of semiprocesses [4] which allows us to investigate the convergence of the attractors of the numerical schemes in which the discretization parameter is not subjected to any law and can tend to zero in an arbitrary manner. This result is applied to the study of the uniform attractor of the explicit finite-difference scheme for the Lorenz system with time-dependent coefficients [5].

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References

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