Structure and regularity of the global attractor of reaction-diffusion equation with non-smooth nonlinear term

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In a bounded domain $\Omega \subset \mathbb{R}^3$ with sufficiently smooth boundary $\partial \Omega$ we consider the problem

$$\begin{cases} u_t - \Delta u + f(u) = h, & x \in \Omega, \ t > 0, \\ u|_{\partial\Omega} = 0, \end{cases}$$
(1)

where $f \in C(\mathbb{R})$ satisfies suitable growth and dissipative conditions, but there is no condition ensuring uniqueness of the Cauchy problem. When the nonlinear term f is smooth and f' satisfies additional assumptions, it is well known [1], that the problem (1) generates semigroup, which has global attractor and it coincides with the unstable set, emanating from the set of stationary points and with stable one as well. In general case (2), when the uniqueness of Cauchy problem is not guaranteed, we have existence of trajectory attractor [2], and existence of global attractor of multivalued semiflow [3]. Our aim is to study the structure of the global attractor in multi-valued case. We prove that the attractor of the multi-valued semiflow generated by all weak solutions of (1) in the phase space $L^2(\Omega)$ is the closure of the union of all stable manifolds of the set of stationary points. Also, for multi-valued semiflow, generated by regular solutions, we prove the existence of global attractor, which is compact in $H_0^1(\Omega)$ and we establish that it is the union of all unstable manifolds of the set of stationary points and of the stable ones as well.

References

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